Seminar on Algorithms and Geometry – Handout 1

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1 Administrative issues

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To receive announcements please join the course's mailing list at:

`www.weizmann.ac.il/mailman/listinfo/2009b-SeminarGeometryAlgs`

This link and other information is available at the course's webpage:

`www.wisdom.weizmann.ac.il/~robi/teaching/2009b-SeminarGeometryAlgs`

Requirements.

- Problem sets (homework), which should be submitted within 2 classes (usually 2 weeks).
- Presenting a paper (in class)
- Writing scribes (1-2 pages summarizing the class).

Reading material. Recommended references and other resources will be posted on the website.

2 Today's topics

- Introduction to metric embeddings – definitions, examples, etc.
- Embedding finite metrics into ℓ_∞ and ℓ_∞^k.

3 Homework

1. Prove that every finite tree metric (shortest-path distances on a graph which is a tree) embeds isometrically into ℓ_1.
2. Prove that for all $1 \le p \le \infty$ and $d \ge 1$, the space ℓ_p^d embeds with distortion that depends only on d (a) into ℓ_2; (b) into ℓ_∞^d. (Note there are infinitely many points.)
3. Prove that (the shortest-path metric of) a star with 3 leaves (as an unweighted graph) does not embed isometrically into ℓ_2.

   Remark: Please write explicitly the distortion lower bound your proof gives, even though there is no need to optimize it.