Seminar on Algorithms and Geometry – Handout 4

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Homework

This problem set refers to the April 16 presentation about probabilistic embedding into trees (Fakcharoenphol, Rao, and Talwar, JCSS 2004). See the webpage for more details.

Recall the main result: Every n-point metric embeds into a distribution over dominating tree metrics with distortion $O(\log n)$. Furthermore, there is an efficient randomized algorithm to produce such a tree metric.

1. Use the main result to prove the following: Every n-point metric embeds into ℓ_1 with distortion $O(\log n)$.

   Note: In class, we discussed Bourgain's theorem which is similar except that the target space is ℓ_2 (which is a stronger statement). We then used such an embedding into ℓ_1 to obtain an $O(\log n)$ approximation algorithm for the sparset-cut problem.

   Hint: Embed a tree metric into ℓ_1.

2. In the derandomization part, the input contains also a weight function $c_{u,v} \geq 0$ (for every $u, v \in V$) and the goal is to find (deterministically) a dominating tree with

$$\sum_{u,v \in V} c_{u,v} d^T(u,v) \leq O(\log n) \sum_{u,v \in V} c_{u,v} d(u,v),$$

   i.e. one tree with small average "stretch" with respect to the weights $c_{u,v}$.

   Use the main result to show a *randomized* algorithm that achieves this goal. Your randomized algorithm should succeed with probability at least 1/2.