1 Today’s topics

- Local partitioning of sparse graphs
- Additive approximation of vertex-cover in planar graphs

Remark: All these results immediately extend to graphs excluding a fixed minor.

2 Open problems

Is there a partition oracle with runtime (or even query complexity) that is polynomial in \(1/\varepsilon\)?

3 Homework

1. Prove that for every \(\varepsilon, d > 0\) there is \(k^* = k^*(\varepsilon, d)\) such that every planar \(G\) with maximum degree \(\leq d\) admits an \((\varepsilon, k^*)\)-partition.

2. Extend the algorithms seen in class (the partition oracle and its usage to approximate vertex-cover) to all planar graphs.
   
   Hint: Use the fact that the average degree is bounded by a constant \(\tilde{d}\), and thus most vertices have small degree as well.

3. Adapt the partition oracle seen in class, so as to bound the worst-case (instead of expected) runtime, as follows: the algorithm (oracle) gets as input also \(0 < \alpha < 1/2\); it is allowed to return “fail”, but this happens with probability \(\leq \alpha\) (over the algorithm’s coins); the runtime (per query) is bounded in terms of \(\varepsilon, d,\) and \(\alpha\).

4. Design an algorithm that tests whether an input graph of maximum degree \(\leq d\) is planar (or \(\varepsilon^*\)-far from planarity).
   
   Hint: apply the above partition oracle (without knowing if the graph is planar), estimate how many edges it removes, and check whether the resulting parts are planar.