Advanced algorithms - handout 1

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The course will cover theoretical design and analysis of algorithms, focusing on computational problems involving combinatorial optimization, such as cuts, flows and distances in graphs. The emphasis will be on modern algorithmic approaches for finding either exact or approximate (near-optimal) solutions, using tools such as mathematical programming, matrix analysis, geometry, sparsification and compression.

Prerequisites: Students are expected to be familiar with algorithms, complexity theory, probability theory, and linear algebra, at an undergraduate level.

Homework assignments are very important to the process of learning, and also effect the final grade to a large extent. Try to do the best you can. They should be handed in within two weeks.

Today's topic - introduction to linear programming: formulating problems as a linear program, standard and canonical forms, basic feasible solutions, polyhedrons.

Homework

1. Consider the problem of minimizing the ratio

$$\frac{c^t x}{f^t x}$$

of two linear functions, subject to all the following constraints:

- $Ax \geq b$,
- $f^t x \geq 1$,
- $-8 \leq c^t x \leq 8$.

Show how linear programming can be used as a subroutine so as to find the optimal solution within any degree of accuracy (the running time may depend on the degree of accuracy required). (Hint: consider the problem of deciding whether the objective function is at most a given value.)

2. Recall that a degenerate solution to an LP in standard form is one in which the number of nonzero variables is less than m. Recall also that a basic feasible solution is determined by a basis of m linearly independent columns for the constraint matrix.

(a) Show that if two bases give the same solution, then this bfs is degenerate. (This was discussed in class, but write down the proof.)

(b) Show that the converse of the above is not always true. That is, write down an LP in standard form that has a degenerate feasible solution, and there is only one basis that gives this degenerate solution.