Advanced Algorithms 2012A – Problem Set 7

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February 4, 2012

Extra credit:

1. Design a variant of Cheeger's inequalities, where the input graph comes with two dinstinguished vertices $s, t \in V$, and we only consider cuts that separate s from t. Specifically, find a value $\lambda_{st}(G)$ that satisfies something like:

   $$\tfrac{1}{2}\lambda_{st}(G) \leq \min_{S:\ |\{s,t\}\cap S|=1} \phi_G(S) \leq \sqrt{2\lambda_{st}(G)}$$

   Remark: This $\lambda_{st}(G)$ need not be an eigenvalue, although it is derived from the (normalized) Laplacian matrix.

   If helpful, you may assume that G is d-regular.

2. Let $G = (V, E)$ be a capacitated graph (i.e., with edge capacities $c_e \geq 0$). Show that the algorithm below outputs a capacitated tree T on the same vertex set V that is *flow-equivalent* to G, in the sense that for all $s, t \in V$, the maximum st-flow in G is equal to that in T. (Notice that T has same vertices, but in general different edges and capacities than G.)

   Algorithm:

   (1) Construct on V a complete graph G', with every edge capacity c'_{st} is equal to $s, t \in V$ are connected by an edge whose capacity c'_{st} is defined to be the maximum st-flow in G.

   (2) Compute a maximum spanning tree T for G' (with respect to the edge capacities c'_e).

   Hint: A maximum spanning tree is analogous to a minimum spanning tree; in particular, it is just a minimum spanning tree for the capacities $-c'_e$.