Advanced Algorithms – Handout 1

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1 Administrative issues

The class will be taught by myself (Zyskind 230, phone x4281, robert.krauthgamer@weizmann.ac.il) and by Uri Feige (Zyskind 251, phone x3364, uriel.feige@weizmann.ac.il). The course website is

http://www.wisdom.weizmann.ac.il/~robi/teaching/AdvAlgs2008

Recommended reading material will be posted on the website.

There will be weekly problem sets (homework), which should be submitted within 2 weeks. There will also be a final exam. These two will be given roughly equal weight in the course grade.

Roughly speaking, the first half of the course will focus on Linear Programming, the second on algorithmic applications of it.

2 Today's topics

- Brief introduction to Linear Programming (LP), including definition, basic terminology, equivalent forms.
- The geometry of LP (the notion of a vertex).

3 Homework

1. Write a linear program that computes the following in a directed graph $G(V, E)$.
   (a) Minimum cost flow: we have to route $d \geq 0$ units of flow (called demand) from source $s \in V$ to sink $t \in V$ respecting the edge capacities $c_{uv} \geq 0$, the cost of routing one unit on edge (u, v) is a_{uv}, and we want to minimize the total cost of the flow.
   (b) Maximum concurrent flow: given k source-sink pairs $(s_1, t_1), \ldots, (s_k, t_k)$, route flow such that the total flow (over all commodities) on edge e respects the edge capacity $c_e \geq 0$, and we want to maximize the minimum, over all $i \in \{1, \ldots, k\}$, of the $s_i - t_i$ flow routed.

2. Consider the problem of minimizing the ratio of two linear functions, as follows.

$$
\begin{aligned}
&\min \frac{c^t x}{d^t x} \\
&\text{subject to} \quad Ax = b \\
&\qquad\qquad\quad x \geq 0 \\
&\qquad\qquad\quad d^t x \geq 1 \\
&\qquad\qquad\quad c^t x \leq 10
\end{aligned}
$$

It is known that $c \geq 0$ (all entries are nonnegative). Show how linear programming can be used to solve this problem within a desired additive accuracy $\varepsilon > 0$. (Hint: consider the decision problem whether the objective is at most a given value B.)

3. Consider the following LP.

$$
\begin{aligned}
&\min x_2 \\
&\text{subject to} \quad x_1 \geq 3 \\
&\qquad\qquad\quad 3x_1 - x_2 \geq -1 \\
&\qquad\qquad\quad x_1 + x_2 \geq 6 \\
&\qquad\qquad\quad -x_1 + 2x_2 \geq -3
\end{aligned}
$$

Draw the constraints and feasible region and find in it the optimum value. Then *prove* that this objective value is indeed the best possible. (Hint: use a combination of linear inequalities.)