# Advanced Algorithms – Handout 2

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## 1 Today's topics

- Continue from last time: The geometry of LP (the notion of a vertex)
- Bases (the notion of a basic feasible solution)
- The Simplex algoithm (a general description)

## 2 Homework

1. Definition: A feasible solution $x$ for a minimization problem is called a *local optimum* if $x$ has an open neighborhood $N_x$ such that no feasible $z \in N_x$ has a smaller objective value than $x$.

   Prove that in every LP, every local optimum is also an optimal solution to the LP (aka global optimum). (Hint: Show first that the set of feasible solutions is convex, i.e. for every two points $x, y \in P$, the entire line segment between them $\{tx + (1-t)y : \ 0 \leq t \leq 1\} \subseteq P$.)

2. Assume the LP $\min\{c^t x : \ Ax = b, \ x \geq 0\}$ has finite value and all its coefficients are integral. Provide an upper bound $M$ on the optimal value of this LP, where $M$ may depend on $n$, on $m$, on $\alpha = \max_{ij} |a_{ij}|$, on $\beta = \max_i |b_i|$, and on $\gamma = \max_j |c_j|$. (Hint: Show first an $\hat{M} = \hat{M}(m, \alpha, \beta)$ such that every basic feasible solution $x$ satifies: $\max_j |x_j| \leq \hat{M}$.)

3. Let $x$ be a basic feasible solution of $P = \{x : \ Ax = b, \ x \geq 0\}$. Show that there exists a cost vector $c$ such that $x$ is the unique optimal solution of the LP $\min\{c^t x : x \in P\}$.