

# The Sixth Israeli Mini-Workshop in Applied Mathematics Dedicated to the Memory of Lee Segel (1932-2005)

Sponsored by: *CENTER FOR ADVANCED STUDIES IN MATHEMATICS (BGU)*

Organizers: Vladimir Gol'dshtein and Boris Zaltzman

We are pleased to invite the Israeli applied math. community to participate in the Sixth Israeli Mini-Workshop in Applied and Computational Mathematics, to be held at the Ben-Gurion University on Monday January 1<sup>st</sup>, 2006.

The idea of the workshop is to create a forum for researchers in applied mathematics, especially younger faculty and students, to get to know each other, and promote discussion and collaboration between researchers from various disciplines and institutions. The workshop will take place in Ben-Gurion University, Mathematical Department (building 58), room -101. See Campus Map at <http://www.cs.bgu.ac.il/information/directions.html>.

We dedicate this mini-workshop to the memory of Lee Segel (February 5, 1932 – January 31, 2005), a great scientist, a wonderful man, one of the leading applied mathematicians and a pioneer of Modern Mathematical Biology.

**To register, please send a short e-mail to the organizers at [boris@bgu.ac.il](mailto:boris@bgu.ac.il). There is no registration fee, but we need to know how many participants to expect.**

The tentative schedule:

Registration: 9.30 till 12.15

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|---------------|--|--|
| 10:30         | Isaak Rubinstein<br>(Ben-Gurion University)  | Opening and Tribute to Lee Segel (1932-2005)   |
| 10:45 - 11:15 | Baruch Meerson<br>(Hebrew University)        | Self-similar asymptotics for a class of Hele-Shaw flows driven solely by surface tension |
| 11:15 - 11:45 | Moshe Goldberg<br>(Technion)                 | Stable finite-difference approximation for parabolic systems                             |
| 11:45 - 12:20 | Coffee break                                 |  |
| 12:20 - 12:50 | Leonid Berezansky<br>(Ben-Gurion University) | Delay differential equations of mathematical biology                                     |

12:45 - 14:20 Lunch break

14:20 - 14:50 Len Pismen  
(Technion)

Wandering patterns in reactive dewetting

14:50 - 15:20 Gadi Fibich  
(Tel-Aviv University)

New singular solutions of the nonlinear  
Shrodinger equation

15:20 - 15:50 Daniel Michelson  
(Weizmann Institute)

Radial asymptotically periodic solutions of the  
Kuramoto-Sivashinsky equation

15:50 - 16:20 Coffee break

16:20 - 16:50 Isaac Goldhirsch  
(Tel-Aviv University)

Strongly inelastic granular gases

16:50 - 17:20 Itzhak Fouxon  
(Hebrew University)

Formation and evolution of particle clusters in  
inelastic gases

17:20 - 17:50 Ehud Yariv  
(Technion)

Electro-magneto-phoresis of slender bodies

## Abstracts:

### Self-similar asymptotics for a class of Hele-Shaw flows driven solely by surface tension

Baruch Meerson (Hebrew University)

I will review the results of our investigations of the dynamics of relaxation, by surface tension, of a family of curved interfaces between an inviscid and viscous fluids in a Hele-Shaw cell. At  $t = 0$  the interface is assumed to be of the form  $|y| = Ax^m$ , where  $A > 0$ ,  $m \geq 0$ , and  $x > 0$ . The case of  $0 < m < 1$  corresponds to a smooth shape,  $m > 1$  corresponds to a cusp, whereas  $m = 1$  corresponds to a wedge. The inviscid fluid tip retreats in the process of relaxation, forming a lobe which size grows with time. Combining analytical and numerical methods we find that, for any  $m$ , the relaxation dynamics exhibit self-similar behavior. For  $m \neq 1$  this behavior arises as an intermediate asymptotics: at late times for  $0 \leq m < 1$ , and at early times for  $m > 1$ . In both cases the retreat distance and the lobe size exhibit power law behaviors in time with different dynamic exponents, uniquely determined by the value of  $m$ . In the special case of  $m = 1$  (the wedge) the similarity is exact and holds for the whole interface at all times  $t > 0$ , while the two dynamic exponents merge to become  $1/3$ . Our numerics show that, surprisingly, when  $m \neq 1$ , the interface shape, rescaled to the local maximum elevation of the interface, is independent of  $m$  in the similarity region. Even more remarkably, the *same* rescaled interface shape emerges in the case of  $m = 1$  in the limit of zero wedge angle. These numerical results are awaiting for a proper theory.

*My collaborators in this series of works are: Omri Gat, Pavel V. Sasorov, and Arkady Vilenkin.*

### Stable Finite-Difference Approximations for Parabolic Systems

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**ABSTRACT.** In this talk we discuss stability criteria for a family of finite-difference approximations to parabolic initial-value problems of the form

$$\frac{\partial \mathbf{u}(x,t)}{\partial t} = \sum_{1 \leq p \leq q \leq s} A_{pq} \frac{\partial^2 \mathbf{u}(x,t)}{\partial x_p \partial x_q} + \sum_{1 \leq p \leq s} B_p \frac{\partial \mathbf{u}(x,t)}{\partial x_p} + C\mathbf{u}(x,t),$$

$$\mathbf{u}(x,0) \in L_2, \quad x = (x_1, \dots, x_s) \in \mathbb{R}^s, \quad 0 \leq t \leq T,$$

where  $A_{pq}$ ,  $B_p$ , and  $C$  are constant matrices. We deal with two cases, the classical case where the leading matrix coefficients  $A_{pq}$  are Hermitian, and the less conventional case where the  $A_{pq}$  are triangular. The second case arise in connection with a number of physical and biological applications; for instance, heat and mass transfer with Soret and Dufour cross-effects, segregation dynamics of granular materials, and interacting population dynamics with dispersion. This is joint work with Anna Pidgirnyak.

### Delay differential equations of mathematical biology

Leonid Berezhansky (Ben-Gurion University)

I will review the results of our study of the delay differential equation models in mathematical biology (e.g., Logistic equation with harvesting; Hematopoiesis equation;

Food-limited equation; Periodic Fox production equation) and address questions of solution oscillation, local and global stability, existence of periodic solutions, extinction and persistence.

### **Wandering patterns in reactive dewetting**

Len Pismen (Technion)

A change of wetting properties of a liquid as a result of a chemical reaction, phase transition or thermal flux, may cause self-propelled motion of droplets observed in a number of experiments. The velocity of motion can be computed analytically using integral conditions that involve both boundary forcing and viscous friction (resolving the latter's singularity at the contact line). Repelling interaction of droplets may lead, depending on the ratio of motion and diffusion times, to mutual scattering or pattern formation, reproducing in a peculiar form related phenomena in activator-inhibitor systems.

### **New singular solutions of the nonlinear Shrodinger equation**

Gadi Fibich (Tel-Aviv University)

The study of singular solutions of the NLS goes back to the 1960s, with applications in nonlinear optics and, more recently, in BEC. Until recently, the only known singular solutions had a self-similar "Gaussian-type" profile that approaches a delta function near the singularity. In this talk I will present new families of singular solutions of the NLS that collapse with a self-similar ring profile, and whose blowup rate is different from the one of the "old" singular solutions. I will also show, both theoretically and experimentally, that these new blowup profiles are attractors for large super-Gaussian initial conditions.

### **Radial asymptotically periodic solutions of the Kuramoto-Sivashinsky equation**

Daniel Michelson (Weizmann Institute)

Rotationally invariant steady solutions of the Kuramoto-Sivashinsky equation in the two space dimensions are studied. It is shown that there exist solutions that approach at infinity the one-dimensional periodic solutions. Both hyperbolic and elliptic periodic solutions are considered.

### **Strongly inelastic granular gases**

Isaac Goldhirsch (Tel-Aviv University)

A novel method for analyzing the Boltzmann equation corresponding to inelastically colliding particles ("grains") has been developed. It is based on several elements, including the observation that the Sonine polynomial expansion for the homogeneous cooling state is asymptotic (though Borel resumable) and the fact that high order expansions in these polynomials may be conveniently carried out by employing the power of symbolic manipulators (such as MAPLE). The results for the transport

coefficients are in excellent agreement with DSMC simulations (unlike all previous results) for all physical values of the coefficient of normal restitution. Further applications and implications will be discussed, time allowing.

### **Formation and evolution of particle clusters in inelastic gases**

Itzhak Fouxon (Hebrew University)

Among the phenomena of structure formation in many-body systems, one of the most interesting is development of clusters of matter from structureless initial states. A most familiar example is gravitational instability, important for the structure formation in the Universe. Another example is clustering instability of a dilute gas of inelastically colliding particles. Such a gas, left alone, behaves opposite to the usual gas – instead of spreading uniformly over the container, it forms regions of high density. Both instabilities are hard to describe beyond the linear theory due to the absence of the regime of weak non-linearity. In this talk we consider the clustering instability in the case of a narrow channel geometry of the container, which confines the gas dynamics to a single spatial dimension. We derive analytic solutions of the fully non-linear system of gas dynamic equations and show that they produce, in a finite time, a new type of hydrodynamic singularity – density spikes. The singularity holds universally for generic initial conditions, and it signals formation of a new phase, where gas particles are densely packed. We continue the solutions beyond the singularity (weak solutions) where they describe a gas flow for which the dilute and the dense phases coexist. The mass of the densely packed regions grows with time. Development of such growing particle clusters can be important for the formation of planet-like bodies in the Universe.

### **Electro-magneto-phoresis of slender bodies**

Ehud Yariv (Technion)

When an insulating particle is placed within a conducting liquid domain which is exposed to uniformly-applied electric and magnetic fields, a rotational Lorentz force density distribution is generated, thereby animating liquid flow. The consequent particle motion is known as Electro-magneto-phoresis (EMP). This mechanism, traditionally employed for impurity extraction in liquid metals, now finds new applications for the manipulation, control and separation of bio- particles.

Symmetry analyses have already demonstrated that the combination of electric and magnetic fields can result in a rich topology of particle motion, unparalleled by other (e.g. phoretic) animation mechanisms. So far, however, analytic solutions for EMP have only been obtained for highly-symmetric particle shapes (spheres and ellipsoids) which do not exhibit the entire richness of particle motions. In this talk, I will describe how slender-body asymptotic theory can be exploited to analyze the EMP of elongated particles which are arbitrarily oriented relative to the externally-imposed fields.

Joint work with Touvia Miloh, Tel Aviv University