Homework #5

Due: May 13

1. For which values of r is the quadratic map a contraction?

$$x_{n+1} = rx_n(1-x_n) = F(x;r), \qquad x \in [0,1], \;\; r > 0. \tag{1}$$

2. Construct numerically, by iterating initial conditions and leaving out the transients (i.e. do not plot the first N iterations for some large number N), the bifurcation diagram for:

   (a) The quadratic map: $x_{n+1} = rx_n(1-x_n) \qquad x_n \in [0,1]$ for $r \in [0,4]$

   (b) The sine map: $x_{n+1} = r\sin \pi x_n \qquad x_n \in [0,1]$ for $r \in [0,1]$

   (c) Let r_n denote the nth period doubling bifurcation in the doubling bifurcation sequence. For both maps, find numerically, for as large an n as you can, the ratio: $\delta_n = \frac{r_n - r_{n-1}}{r_{n+1}-r_n}$. Can you see convergence to the Universal Feigenbaum constant $\delta = 4.669201..$? (bonus: derive more sophisticated ways to find δ_n, read about it).

3. Let Σ_N consist of all sequences of natural numbers $\{0,1,2,..,N-1\}$. Let σ denote the shift map on these sequences.

   (a) Find $CardPer_k(\sigma)$: the number of the periodic points of σ of period k.

   (b) Show that σ has a dense orbit.

   (c) Consider the map: $x_{n+1} = 3x_n$ mod 1. Prove that the map is chaotic (hint: use the symbolic dynamics on Σ_3). Prove that the middle-third Cantor set Λ is invariant under the map and that the map has a dense orbit on Λ (hint: use the subset of Σ_3 of sequences containing only the symbols $\{0,2\}$).

4. Bonus: find a paper in your field of interest in which period doubling bifurcation plays a role. Write the equations leading to the bifurcation, and describe the implications to the specific problem.