Homework #7

Due: June 10

1. Classify the dynamics of the following ODEs $\dot{x} = Ax$ for the following matrices A. Sketch the phase portrait (qualitatively, not numerically).

   (a) $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, (b) $A = \begin{pmatrix} -3 & 2 \\ -2 & 1 \end{pmatrix}$, (c) $A = \begin{pmatrix} 1 & -2 \\ 4 & -3 \end{pmatrix}$,

   (d) $A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$, (e) $A = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix}$, (f) $A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$

2. (Meiss 2.14)

Solve the initial value problem $\frac{dx}{dt} = Ax, \quad x(0) = x_o$ with

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & -4 \\ 1 & 4 & 2 \end{pmatrix}$$

and $x_o = (1, 1, 0)^T$.

3. In each of the problems below determine whether the fixed point in the origin is stable, asymptotically stable or unstable by constructing a suitable Lyapunov function of the form $ax^2 + cy^2$ where a and c are to be determined

$$\dot{x} = -x^3 + xy^2, \quad \dot{y} = -2x^2y - y^3,$$
$$\dot{x} = -\frac{1}{2}x^3 + 2xy^2, \quad \dot{y} = -y^3,$$
$$\dot{x} = -x^3 + 2y^3, \quad \dot{y} = -2xy^2,$$
$$\dot{x} = x^3 - y^3, \quad \dot{y} = 2xy^2 + 4x^2y + 2y^3.$$

4. Consider the system of equations

$$\dot{x} = y - xf(x,y), \qquad \dot{y} = -x - yf(x,y),$$

   where f is continuous and has continuous first partial derivatives. Show that if $f(x,y) > 0$ in some neighborhood of the origin, then the origin is an asymptotically stable fixed point, and if $f(x,y) < 0$ in some neighborhood of the origin, then the origin is an unstable fixed point.