Homework #10

Due: July 3

1. Consider the stable and unstable manifolds of a hyperbolic fixed point of a C^r vector field in R^n (for example $n = 2$ and $n = 3$):

$$\frac{dx}{dt} = f(x), \quad x \in R^n$$

   (a) Can the stable (respectively, unstable) manifold intersect itself? Why?

   (b) Can the stable (respectively, unstable) manifold intersect the stable (respectively, unstable) manifold of a different fixed point? Why?

   (c) Can the stable manifold intersect the unstable manifold? If so, what is the nature of the intersection?

   (d) Can the stable (respectively, unstable) manifold intersect a periodic orbit? Why?

2. Consider questions 1a-d for the stable and unstable manifolds of a hyperbolic fixed point of a C^r diffeomorphism in R^2 (for example, the Henon map).

3. Consider the periodically vertically forced and damped pendulum equation:

$$\frac{dq}{dt} = p, \quad \frac{dp}{dt} = -\sin(q)(1+\varepsilon\sin(\omega t)) - \delta p \tag{1}$$

   (a) Compute the Melnikov function and explain your results in view of questions 1-2 (it is sufficient to find the Melnikov function up to quadratures).

   (b) Integrate numerically the equations and present your results by projections to various spaces and by looking at the time T Poincare map. Explain your results.