Homework #9
Due: June 26

1. Ex 12 of Chapter 4 of Meiss book:

   12. Assume that the flow $\varphi_t : A \to A$ is conjugate to the flow $\psi_t : B \to B$ with conjugacy $h : A \to B$.

      (a) Show that if $\omega(x)$ is the omega limit set for $x \in A$ under φ, then $h(\omega(h^{-1}(y)))$ is the omega limit set for $y = h(x) \in B$ under ψ.

      (b) Show that if Λ is an invariant set for φ, then $h(\Lambda)$ is an invariant set for ψ.

      (c) Show that if $W^s(\Lambda)$ is the basin of Λ, then $h(W^s(\Lambda))$ is the basin of $h(\Lambda)$.

      (d) Show that if Λ is an attractor, then so is $h(\Lambda)$.

2. Consider the neutrophils-G-CSF model (g- represents the G-CSF levels in the blood and n - the neutrophils - one kind of white blood cells):

$$\frac{dg}{d\tau} = \frac{a_1}{n+0.1} - \left(a_2 + \frac{a_3 \cdot n}{n+0.1}\right) \cdot g,$$
$$\frac{dn}{d\tau} = a_4 \cdot \left(\frac{1+0.01 \cdot a_5 \cdot g}{1+0.01 \cdot g}\right) - n.$$

   where $a_1 = 0.5, a_2 = 1.4, a_3 = 0.5, a_4 = 1, a_5 = 6$

   (a) Find the qualitative behavior of the null-clines: show that they are monotone and can cross only once. Discuss the robustness of this statement as the parameters are varied.

   (b) Find the fixed point and its stability - this fixed point corresponds to homeostasis.

   (c) Prove that there can be no limit cycles in this system.

   (d) Bonus: See Shochat et al. 2007 and 2008 and Malka et al. 2012 for axiomatic model construction, motivation, analysis and implications.

   (e) Bonus: Find how circadian forced oscillations in G-CSF concentration change the systems behavior.