Homework #2

Due: November 10 (You may use the programs on the course web page)

1. Consider the initial value problem

$$\dot{y} = \lambda y, \quad y(0) = 1, \quad \lambda = -50.$$

For $t = [0,1]$ display (y,t) graph for both the analytic solution and the numerical solution by solving with:

   (a) FE and RK4 methods for different time steps h. What should be h for the method to converge?

   (b) "ode45" and "ode15s" functions in Matlab (or the equivalent programs in Python). Try to increase $|\lambda|$ and t and describe the difference between them.

2. Generally, the pendulum motion depends on the length L of the pendulum, the mass m of the bob, the gravitational constant, g, the initial location, $x(0) = l$, and the initial velocity, $\dot{x}(0) = v$.

   (a) Show that it is impossible, on dimensional grounds, that the period T depends only on the length L of the pendulum and the mass m of the bob, that is, one cannot find a function f such that $f(T,L,m) = 0$

   (b) It is observed that for sufficiently small vibrations, to a good approximation, the period of the motion does not depend on l and v. For a pendulum executing small vibrations, show, by dimensional analysis, that the assumption that T depends only on m, L, and g, leads to the relation $T = k\sqrt{L/g}$, where k is a unit-less constant.

   (c) Suppose that the pendulum is pulled out farther, so that its amplitude of oscillation is no longer small. How many non-dimensional variables the modified model has? Write the functional dependence of T on the other parameters and discuss your results consistency with the limit of small oscillations.

   (d) Non-dimensionalize the pendulum equations and solve them numerically for several typical initial conditions with FE, RK4 and the matlab program ode45. Present your results in phase space and as time plots. Explain your results - explore the period of your solutions as you change your initial conditions and explore how to get all types of typical solutions. Check whether your code preserves the total energy (see how this depends on changing the error in "options").

3. List at least three (and up to 5) most important concepts learned in your reading (Chapter 1 of Meiss and/or additional material, see below).
4. List at least one result/notion which you find significant to you (related to area of your interest/difficult/non-intuitive/other).