Homework #7

Due: December 2

1. For which values of r is the quadratic map a contraction?

$$x_{n+1} = r x_n (1 - x_n) = F(x; r), \qquad x \in [0,1], \;\; r > 0. \tag{1}$$

2. Let Σ_N consist of all sequences of natural numbers $\{0,1,2,..,N-1\}$. Let σ denote the shift map on these sequences.

   (a) Find $CardPer_k(\sigma)$: the number of the periodic points of σ of period k.

   (b) Show that σ has a dense orbit.

   (c) Consider the map: $x_{n+1} = 3x_n \bmod 1$. Prove that the map is chaotic (hint: use the symbolic dynamics on Σ_3). Prove that the middle-third Cantor set Λ is invariant under the map and that the map has a dense orbit on Λ (hint: use the subset of Σ_3 of sequences containing only the symbols $\{0,2\}$).

3. Bonus: find a paper in your field of interest in which symbolic dynamics is used and explain what you found.