Homework #7
Due: December 2

1. For which values of $r$ is the quadratic map a contraction?

\[ x_{n+1} = rx_n(1 - x_n) = F(x; r), \quad x \in [0, 1], \quad r > 0. \]  

(1)

2. Let $\Sigma_N$ consist of all sequences of natural numbers $\{0, 1, 2, \ldots, N-1\}$. Let $\sigma$ denote the shift map on these sequences.

   (a) Find $\text{CardPer}_k(\sigma)$: the number of the periodic points of $\sigma$ of period $k$.

   (b) Show that $\sigma$ has a dense orbit.

   (c) Consider the map: $x_{n+1} = 3x_n \mod 1$. Prove that the map is chaotic (hint: use the symbolic dynamics on $\Sigma_3$). Prove that the middle-third Cantor set $\Lambda$ is invariant under the map and that the map has a dense orbit on $\Lambda$ (hint: use the subset of $\Sigma_3$ of sequences containing only the symbols $\{0, 2\}$).

3. Bonus: find a paper in your field of interest in which symbolic dynamics is used and explain what you found.