Homework #8b - 1d dynamics
Due: December 9

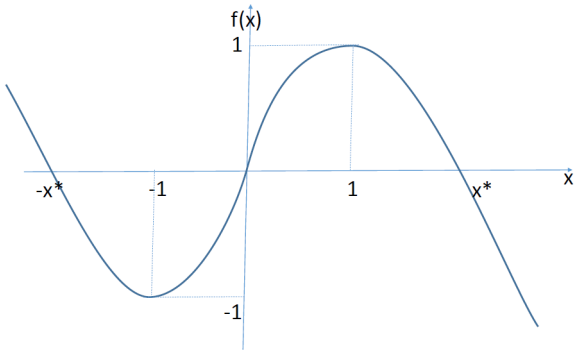

Figure 1: Assume f is monotone decreasing for all $|x|$ larger than x^*, and that f is at least C^2 smooth.

1. Given the graph in Figure 1 of the smooth function $f(x)$, describe the corresponding dynamics of the 1-d flow

$$\frac{dx}{dt} = f(x), \quad x \in \mathbb{R} \tag{1}$$

and the 1-d map

$$y_{n+1} = f(y_n), \qquad y_0 \in [0,1] \tag{2}$$

In particular:

   (a) Identify graphically the fixed points of the flow and of the map and find their stability.

   (b) Describe all possible types of solutions and plot all the typical behavior in the phase space and the time plots.

2. Let $g(x;a,b,c,d) = a + b f(cx-d)$, where $a,b,c,d \geqslant 0$ (these are the functions produced by $f(x)$ by rescaling the axes and shifting the origin).

   (a) Plot $g(x;a,b,c,d)$ for some typical non-zero values of the parameters.

(b) Consider the 4 parameter family of flows:

$$\frac{dx}{dt} = g(x;a,b,c,d), \quad x \in \mathbb{R} \tag{3}$$

Explain when the flow produced by (1) and (3) are equivalent and in what sense. Explain which bifurcations are produced and find the bifurcation sets. Draw the bifurcation diagram in terms of one of the non-degenerate parameters. (you may phrase the results as lemmas using the theorems we learned in class).

(c) Consider now the 1 parameter family of 1-d maps on the line:

$$y_{n+1} = g(y_n;0,b,1,0) = bf(y_n), \qquad y_0 \in \mathbb{R} \tag{4}$$

i. What is the dynamics for sufficiently small b?

ii. What is the dynamics at $b = x^*$?

iii. For $b > x^*$, define $\Lambda_+ = \{y_0 | y_j \in [0,x^*] \text{ for all } j \geqslant 0\}$. Describe the dynamics on Λ_+ for sufficiently large b (define the needed condition on b).

iv. **Bonus:** discuss other interesting cases, in particular, explain similarities and differences between the dynamics and the bifurcations under $g(y_n;a,b,c,d)$ and those of the quadratic map (warning: this could evolve to become an endless task.. do it only after you are done with the exam, for your own entertainment and on your own risk..).

3. Summarize, in your own words, 3 of the most important results you learned about 1d dynamics