Homework #10

Due: December 23

1. Meiss Book, Chapter 5, solve one of the two exercises 8,9 and sketch the main steps for solving the other:

   8. Consider the system on $\mathbb{R}^2$ given by

   $$\dot{x} = -x + xy,$$
   $$\dot{y} = 2y + x^2.$$

   (a) Find E^s and E^u for the fixed point $(0,0)$.

   (b) Construct successive approximations $(x_i(t), y_i(t))$, $i = 1,2$, to the stable manifold $W^s(0,0)$ by applying the operator T, (5.17), to the initial guess $(x_o(t), y_o(t)) = (0,0)$.

   (c) Compare the approximations in (b) with a power series expansions for the stable and unstable manifolds using the techniques of §5.6.

   (d) Using your favorite software, plot the functions you constructed and some numerical solutions of the differential equations. Compare the manifolds that you compute with the solutions.

   9. Consider the system

   $$\dot{x} = x^3 - 2xy,$$
   $$\dot{y} = -y + x^2.$$

   (a) Find the first few terms in the power series expansion for the stable and center manifolds of the origin.

   (b) Study the reduced dynamics on the center manifold. Show that $x(t) \sim t^{-1/2}$ as $t \to \infty$. Classify the equilibrium.

   (c) Compare your analytical expression with numerical orbits generated by your favorite software package.

2. Bonus: find a paper in your field of interest in which center and/ or stable and unstable manifolds play a role. Explain what was the main use of this tool in the paper.