Temporal Factorization Vs. Spatial Factorization

Lihi Zelnik-Manor[1] and Michal Irani[2]

[1] California Institute of Technology, Pasadena CA, USA,
lihi@caltech.edu,
WWW home page: http://www.vision.caltech.edu/lihi
[2] Weizmann Institute of Science, Rehovot, Israel

Abstract. The traditional subspace-based approaches to segmentation (often referred to as multi-body factorization approaches) provide spatial clustering/segmentation by grouping together *points* moving with consistent *motions*. We are exploring a dual approach to factorization, i.e., obtaining temporal clustering/segmentation by grouping together *frames* capturing consistent *shapes*. Temporal cuts are thus detected at non-rigid changes in the shape of the scene/object. In addition it provides a clustering of the frames with consistent shape (but not necessarily same motion). For example, in a sequence showing a face which appears serious at some frames, and is smiling in other frames, all the "serious expression" frames will be grouped together and separated from all the "smile" frames which will be classified as a second group, even though the head may meanwhile undergo various random motions.

1 Introduction

The traditional subspace-based approaches to multi-body segmentation (e.g., [6, 7, 9]) provide spatial clustering/segmentation by grouping *points* moving with consistent *motions*. This is done by grouping *columns* of the correspondence matrix of [17] (we review the definition in Section 1.1). In this work we show that to obtain *temporal* grouping of frames we cluster the *rows* of the same correspondence matrix instead of its columns. We show that this provides grouping of *frames* capturing consistent *shapes*, but not necessarily same motion. We further show that, to obtain such shape-based clustering of frames we need *not* develop any new segmentation/clustering scheme. We can use any of the existing algorithms suggested for clustering points (e.g., [6, 7, 9]). But, instead of applying them to the correspondence matrix as is, we apply them to its transpose.

Note, that spatial "multi-body factorization" [6, 7, 9] usually provides a highly sparse segmentation since commonly the number of points which can be tracked reliably along the sequence is low. Dense spatial segmentation requires dense optical flow estimation (e.g., [11]). In contrast, a small number of tracked points suffices to obtain a dense temporal clustering of frames, i.e., a classification of *all* the frames in the video clip. Furthermore, the dimensionality of the data, which is one of the major difficulties in spatial multi-body factorization, is significantly smaller for temporal segmentation. To obtain dense *spatial* factorization of the

entire image (e.g., [11]), the number of points equals the number of pixels in the image which can be extremely large (hundreds of thousands of pixels). This is *not* the case with *temporal* factorization. The number of frames in the video clip is usually only tens or hundreds of frames, and therefore the temporal factorization is *not* time consuming.

The standard approaches to temporal segmentation cut the video sequence into "scenes" or "shots", mainly by drastic changes in image appearance (e.g., [20, 16, 12]). Other approaches are behavior based (e.g., [19, 15]) and segment the video into sub-sequences capturing different events or actions. The approach suggested here is fundamentally different and provides a temporal segmentation and clustering of frames which is based on non-rigid changes in shape. For example, in a sequence showing a face at some frames serious and in other frames smiling, all the "serious expression" frames will be grouped together and separated from all the "smile" frames which will be classified as a second group, even though the head may meanwhile undergo various random motions.

Our way of formulating the problem provides a unified framework for analyzing and comparing a number of previously developed independent methods. This new view of previous work is described in Section 4. For example, we show that the technique of Rui & Ananadan [15] can be reformulated in terms of the factorization approach. Our analysis illustrates that their approach will detect cuts at large changes in motion, whereas we detect cuts at non-rigid shape changes. In a different work, Rao & Shah [14] suggested a view-invariant recognition method for complex hand movements. In Section 4 we show that the similarity constraint they use for *matching* shapes is equivalent to the one we use for *separating* between shapes.

We start by defining notations and reviewing the background to the multi-body factorization approach in Section 1.1. In Section 2 we present our approach to temporal factorization of shape and in Section 3 we explore its similarities to and differences from the standard spatial factorization of motion. As mentioned above, we review some related works in Section 4 and summarize in Section 5.

1.1 Background on Factorization Methods

Let $I_1, \ldots, I_F$ denote a sequence of F frames with N points tracked along the sequence. Let (x_i^f, y_i^f) denote the coordinates of pixel (x_i, y_i) in frame I_f ($i = 1, \ldots, N$, $f = 1, \ldots, F$). Let X and Y denote two $F \times N$ matrices constructed from the image coordinates of all the points across all frames:

$$X = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_N^1 \\ x_1^2 & x_2^2 & \cdots & x_N^2 \\ & & \vdots & \\ x_1^F & x_2^F & \cdots & x_N^F \end{bmatrix} \qquad Y = \begin{bmatrix} y_1^1 & y_2^1 & \cdots & y_N^1 \\ y_1^2 & y_2^2 & \cdots & y_N^2 \\ & & \vdots & \\ y_1^F & y_2^F & \cdots & y_N^F \end{bmatrix} \tag{1}$$

Each row in these matrices corresponds to a single frame, and each column corresponds to a single point. Stacking the matrices X and Y of Eq. (1) vertically

results in a $2F \times N$ "correspondence matrix" $W = \left[\frac{X}{Y}\right]$. It has been previously shown that under various camera and scene models [17, 8, 5] the correspondence matrix W of a single object can be factorized into motion and shape matrices: $W = MS$ (where M and S are low dimensional). When the scene contains multiple objects (see [6, 7]) we still obtain a factorization into motion and shape matrices $W = MS$, where M is a matrix containing the motions of all objects and S is a block-diagonal matrix containing the shape information of all objects.

2 Temporal Factorization

The traditional subspace-based approaches to multi-body segmentation (e.g., [6, 7, 9]) provide *spatial* clustering of image points by grouping *columns* of the correspondence matrix $W = MS$. Note, that in the correspondence matrix W every column corresponds to a point and every row corresponds to a frame. Thus, to obtain *temporal* clustering of frames we will apply clustering to the *rows* of W instead of its columns. In this section we discuss the physical meaning of this temporal clustering of frames and suggest methods for obtaining it.

When factoring the correspondence matrix W into motion and shape, the columns of the motion matrix M span the columns of W and the rows of the shape matrix S span the rows of W. Hence, clustering the *columns* of W into independent linear subspaces will group together points which share the same *motion*. Equivalently, clustering the *rows* of the correspondence matrix W will group frames which share the same *shape*. Luckily, to obtain such row-based segmentation/clustering we need *not* develop any new segmentation/clustering scheme. We can use any of the existing algorithms suggested for segmenting/clustering columns (e.g., [6, 7, 9]). But, instead of applying them to W, we will apply them to W^T. We next show why this is true.

When the scene contains multiple (K) objects moving with independent motions, and the columns of W are sorted to objects, then according to [6] the resulting shape matrix has a block diagonal structure:

$$W = [W_1, \ldots, W_K] = [M_1, \ldots, M_K] \begin{bmatrix} S_1 & & 0 \\ & \ddots & \\ 0 & & S_K \end{bmatrix} \tag{2}$$

where $W_i = M_i S_i$ is the correspondence matrix of the i-th object, with motion M_i and shape S_i. The correct permutation and grouping of columns of W into $W_1, \ldots, W_K$ to obtain the desired separation into independently moving objects was accordingly recovered [6, 7] by seeking a block-diagonal structure for the shape matrix S. In other words, to obtain spatial segmentation of points we group the columns of W into independent linear subspaces by assuming that W can be factored into a product of two matrices, where the matrix on the right has a block diagonal form.

Now, taking the dual approach: When the sequence includes non-rigid shape changes (Q independent shapes) and the *rows* of W are sorted according to

shape, then the resulting *motion* matrix has a block diagonal structure:

$$W = \begin{bmatrix} \tilde{W}_1 \\ \vdots \\ \tilde{W}_Q \end{bmatrix} = \begin{bmatrix} \tilde{M}_1 & & 0 \\ & \ddots & \\ 0 & & \tilde{M}_Q \end{bmatrix} \begin{bmatrix} \tilde{S}_1 \\ \vdots \\ \tilde{S}_Q \end{bmatrix} \tag{3}$$

The permutation and grouping of *rows* of W into $\tilde{W}_1, \ldots, \tilde{W}_Q$ to obtain the desired separation into frames capturing independent *shapes* can therefore be obtained by seeking a block-diagonal structure for the *motion matrix* M.

Note, however, that if we now take the transpose of W we get:

$$W^T = [\tilde{W}_1^T, \ldots, \tilde{W}_Q^T] = [\tilde{S}_1^T, \ldots, \tilde{S}_Q^T] \begin{bmatrix} \tilde{M}_1^T & & 0 \\ & \ddots & \\ 0 & & \tilde{M}_Q^T \end{bmatrix} \tag{4}$$

That is, the matrix W^T can be factored into a product of two matrices where the matrix on the right is block diagonal. This is equivalent to the assumption made in the factorization of W to obtain column clustering. Thus, we can use any of the algorithms suggested for segmenting/clustering columns (e.g., [6, 7, 9]), however, instead of applying them to W we will apply them to W^T. Our approach to subspace-based temporal clustering/factorization can therefore be summarized as follows:

Given a video clip of a dynamic scene:

1. Track reliable feature points along the entire sequence.
2. Place each trajectory into a column vector and construct the correspondence matrix $W = \left[\frac{X}{Y}\right]$ (see Eq. (1))
3. Apply any of the existing algorithms for column clustering (e.g., "multi-body factorization" of [6, 7, 9]), but to the matrix W^T (instead of W).

Note, that when we say "independent shapes" we refer to independence between *rows* of different shape matrices (and *not* between columns/points). Independence between rows of two shape matrices occurs when at least part of the columns in those matrices are different. Recall, that the matrix S corresponding to a rigid set of points is a $4 \times N$ matrix where each column holds the homogeneous coordinates $[X, Y, Z, 1]^T$ of a 3D point. Rigid shape changes can be viewed as the same set of points undergoing a different rigid motion, and therefore still have the same shape. However, *non-rigid* shape changes imply that some of the points move differently than others, i.e., some of the columns of the shape matrix change differently than others. This will lead to a different shape matrix and thus to assigning these frames to separate temporal clusters. Since every $4 \times N$ shape matrix has a row of 1's there is always partial linear dependence between shape matrices. To overcome that, we can use the Tomasi-Kanade [17] approach for removing the translational component by centering the centroid of

the tracked points. Then the row of 1's is eliminated from the shape matrix, and we obtain full linear independence. Alternatively, some of the previously suggested approaches for sub-space segmentation can handle partial dependencies. In particular, we used the spectral clustering approach suggested in [13].

To illustrate this, Fig. 1 displays frames from a sequence showing a hand first open and then closed, while rotating and translating. As long as the hand is open, i.e., it's shape is not changing, the rows in the matrix W will correspond to the same shape $\tilde{S}_{OPEN}$. However, the closing of the fingers implies a different shape of the object which cannot be represented as a rigid motion change. Instead we will obtain a new shape matrix $\tilde{S}_{CLOSE}$ so that: $W = \begin{bmatrix} \tilde{M}_{OPEN} & 0 \\ 0 & \tilde{M}_{CLOSE} \end{bmatrix} \begin{bmatrix} \tilde{S}_{OPEN} \\ \tilde{S}_{CLOSE} \end{bmatrix}$. Grouping the rows of W is expected to group all the "OPEN" frames into one cluster and all the "CLOSE" frames into a separate cluster.

Fig. 2 shows this on a real video sequence. It further illustrates the difference between spatial segmentation/grouping of points based on *motion (column)* clustering, and temporal segmentation/grouping of frames based on *shape (row)* clustering. The sequence shows a hand opening and closing the fingers repeatedly. Feature points on the moving fingers were tracked along the sequence using the KLT tracker [10, 1] and used to construct the correspondence matrix W. Factoring the rows of W (i.e., the columns of W^T) into two clusters resulted in temporal shape-based segmentation of frames: It grouped together all the frames with fingers stretched open into one cluster, and all the frames with fingers folded into a second cluster (see Figs. 2.a,b,c). In contrast, applying the segmentation to the columns of W resulted in spatial motion-based segmentation of points into independently moving objects: It grouped into one cluster the points on the fingers which moved mostly horizontally, and grouped into a second cluster points on the thumb which moved mostly vertically, (see Fig. 2.d). The palm of the hand was stationary and hence was ignored.

Fig. 3 displays another example of shape-based temporal segmentation. The video clip was taken from the movie "Lord of the Rings - Fellowship of the Ring", and shows two hobbits first relaxed and then screaming. Feature points were tracked along the sequence using the KLT tracker [10, 1] and used to construct the correspondence matrix W. Grouping the rows of W (columns of W^T) into two clusters detected the cut between the two expressions and grouped together all the "calm" frames separately from the "screaming" frames.

3 Comparing Temporal and Spatial Factorization

In this section we explore the major similarities and differences between the common motion based spatial factorization and our suggested approach to shape based temporal factorization.

Data dimensionality: One of the major difficulties in the multi-body factorization approach is the dimensionality of the data. As was shown by Weiss [18], the method of Costeira & Kanade [6] to multi-body segmentation is equivalent

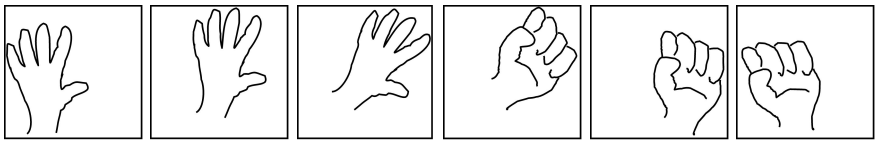


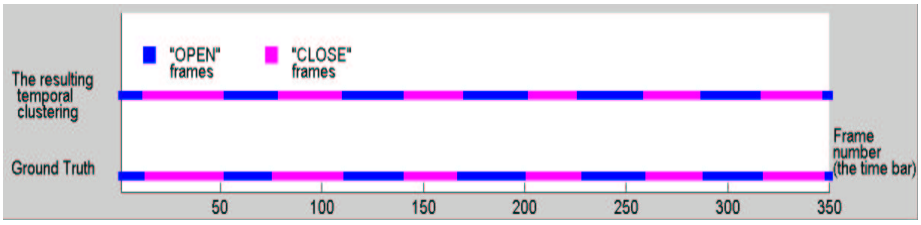
Fig. 1. An illustration of a hand rotating and translating while opening and closing its fingers. *Frames (a),(b) and (c) capture the same shape (open hand) undergoing different rigid motion transformations. The closing of the fingers between frames (c) and (d) generates a new shape, independent of the previous one. The transformations between frames (d),(e),(f) can again be viewed as the same shape (closed hand) undergoing rigid motions.*

to applying spectral clustering to W^TW, which is an $N \times N$ matrix (N being the number of points). If the number of points is large, then this is a very large matrix. Finding the eigenvectors of such a matrix (which is the heart of spectral clustering) is therefore extremely time consuming. To obtain dense *spatial* factorization of the entire image (e.g., [11]), the number of points N equals the number of pixels in the image which can be extremely large (hundreds of thousands of pixels).

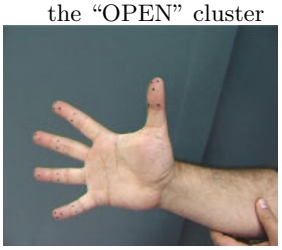
However, this is *not* the case with *temporal* factorization. As explained in Section 2, to obtain temporal factorization of W, we apply the same algorithms suggested for spatial segmentation, but to W^T. In other words, this is equivalent to applying spectral clustering [18] to the matrix WW^T (instead of W^TW). The dimension of WW^T is $2F \times 2F$, where F is the number of frames in the video clip. Since $F << N$ (F is usually only tens or hundreds of frames) , WW^T is thus a small matrix, and therefore the temporal factorization is *not* time consuming. Furthermore, while *dense* spatial factorization requires dense flow estimation, dense temporal factorization can be obtained even if only a sparse set of reliable feature points are tracked over time. This is further explained next.

Tracking sparse points vs. dense optical flow estimation: Each column of W contains the trajectory of a single point over time. The data in the matrix W can be obtained either by tracking a sparse set of reliable points or by dense optical flow estimation. Since the spatial "multi-body factorization" clusters the *columns* of W, it will therefore classify only the points which have been tracked. Thus, when only a small set of reliable points is tracked, the resulting spatial segmentation of the image is sparse. Dense spatial segmentation of the image domain requires dense optical flow estimation. This, however, is not the case with temporal segmentation. Since our temporal factorization clusters the *rows* of W, there is no need to obtain data for all the points in the sequence. A sparse set of reliable points tracked through all the frames suffices for *dense* temporal factorization. This is because the number of columns in W need not be large in order to obtain good row clustering. Results of temporal factorization using a small number of point tracks are shown in Figs. 2 and 3. In Fig. 4 we used dense optical flow measurements to show validity of the approach to both ways

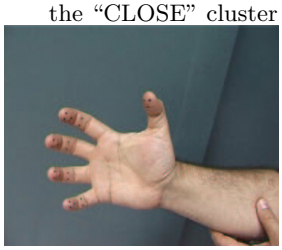
(a) Temporal factorization result:



(b) Example frame from the "OPEN" cluster



(c) Example frame from the "CLOSE" cluster



(d) Spatial factorization result

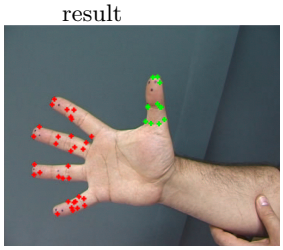


Fig. 2. Temporal vs. spatial clustering. *(a) Results of temporal factorization (on the rows of W = the columns of W^T) applied to a sequence showing a hand closing and opening the fingers repeatedly. Setting the number of clusters to 2 resulted in grouping all frames with fingers open into one cluster (marked in blue on the time bar) and all frames with fingers folded into a second cluster (marked in magenta on the time bar). Ground truth values, obtained manually, are shown for comparison. (b),(c) Example frames of the two temporal clusters. (d) Result of spatial factorization (on the columns of W) applied to the same sequence and the same tracked points. This grouped together all the points on the fingers (marked in red), which move mostly horizontally, and classified into a second cluster points on the thumb (marked in green) which move mostly vertically. Note, that since only sparse feature points were tracked along the sequence, the resulting spatial segmentation is highly sparse, whereas the resulting temporal factorization is dense (i.e., all the frames in the video sequence are classified) even though only a sparse set of points is used. Video can be found at http://www.vision.caltech.edu/lihi/Demos/TemporalFactorization.html*

of obtaining data. Note, however, that even-though N (the number of points) is large when using optical flow, the computational complexity of the temporal factorization is still low, since the size of WW^T is independent of N (it depends only on the number of frames F).

Segmentation of $\left[\frac{X}{Y}\right]$ Vs. $[X,Y]$: Let $W_v = \left[\frac{X}{Y}\right]$ and $W_h = [X,Y]$ where the subscript v stands for vertical stacking of X and Y whereas the subscript h stands for horizontal stacking of X and Y. The common approaches to multi-body factorization (e.g., [6, 7, 9]) selected carefully tracked feature points, constructed the $W_v = \left[\frac{X}{Y}\right]$ matrix and clustered its columns. In this matrix each point has a single corresponding column, and each frame has two corresponding rows. Machline et al. [11] suggested applying multi-body factorization instead

to the columns of $W_h = [X, Y]$. This allows to introduce directional uncertainty into the segmentation process, and thus enables dense factorization using unreliable points as well (i.e., dense flow). In this matrix, (i.e., W_h) each point has two corresponding columns whereas each frame has a single corresponding row. Thus, when clustering frames (rows) using temporal factorization it is simpler to use the matrix $W_h = [X, Y]$. Note, that when switching from $W_v = \left[\frac{X}{Y}\right]$ to $W_h = [X, Y]$ the motion matrix completely changes its structure whereas the shape matrix does not. Thus, in spatial multi-body factorization, which is motion based, there is an inherent difference between the two approaches that leads to a different spatial segmentation when using $W_h = [X, Y]$ vs. $W_v = \left[\frac{X}{Y}\right]$ (see [11]). In contrast, the temporal factorization depends only on shape, thus applying temporal clustering either to $W_v = \left[\frac{X}{Y}\right]$ or to $W_h = [X, Y]$ will provide the same results. For simplicity we used the $W_h = [X, Y]$ matrix for temporal factorization and the $W_v = \left[\frac{X}{Y}\right]$ matrix for spatial clustering of points.

Example: Fig. 4 shows an example of shape vs. motion segmentation using dense optical flow estimation instead of sparse tracking data. The video clip was taken from the movie "Brave Heart", and shows the actor (Mel Gibson) first serious and then smiling while moving his head. The frame-to-frame optical flow was estimated using the robust optical flow estimation software of Michael Black [2] which is described in [4, 3]. The frame-to-frame optical flow fields were composed over time to obtain flow-fields of all frames in the video clip relative to a single reference frame. These flow-fields were then organized in row vectors and stacked to provide the matrix $W_h = [X, Y]$. Applying spectral clustering to the rows of W_h (i.e., applying factorization to the $F \times F$ matrix $W_h W_h^T$) separated the frames into two clusters: one cluster containing all the "smile" frames, and the other cluster containing all the "serious" frames (see Figs. 4.a,b). For comparison, applying the same clustering algorithm to the columns of W_v (i.e., applying multi-body factorization to the $N \times N$ matrix $W_v^T W_v$) separated between regions with different motions (see Fig. 4.c).

Summary: For further clarification, we summarize in table 1 the observations made in Sections 2 and 3. This provides a summary of the comparison between spatial and temporal factorizations.

	Spatial Factorization	Temporal Factorization
Apply clustering to	$W^T W$	$W W^T$
Data dimensionality	$N \times N$	$F \times F$
Data type	Points (columns)	Frames (rows)
Cluster by	Consistent motions	Consistent shapes
Sparse input	Sparse spatial segmentation	Dense temporal segmentation
Dense input	Dense spatial segmentation	Dense temporal segmentation

Table 1. Comparison summary of spatial factorization vs. temporal factorization

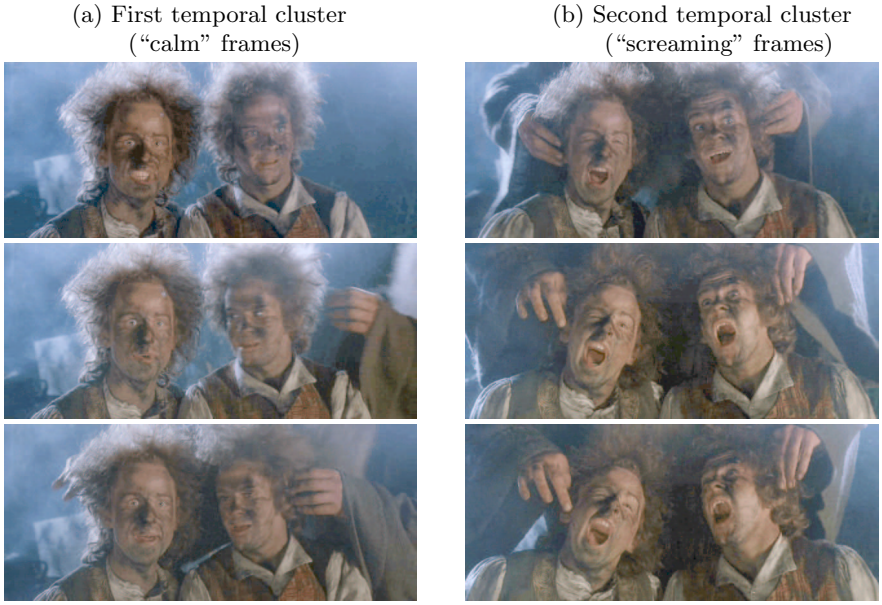


Fig. 3. Temporal clustering of frames. *Results of temporal factorization (into 2 clusters) applied to a video clip taken from the movie "Lord of the Rings - Fellowship of the Ring". The clip shows two hobbits first calm and then screaming. The shape-based temporal factorization detected the cut between the two expressions and grouped together all the "calm" frames (some example frames are shown in column (a)) separately from all the "scream" frames (some example frames are shown in column (b)). Video can be found at http://www.vision.caltech.edu/lihi/Demos/TemporalFactorization.html*

4 A New View on Previous Work

In this section we show that our way of formulating the temporal factorization problem provides a unified framework for analyzing and comparing a number of previously developed independent methods.

The most related work to ours is that of Rui & Ananadan [15] who used changes in the frame-to-frame optical flow field to segment activities into their fragments. Rui & Ananadan [15] estimated the optical flow field between each pair of consecutive frames and stacked those into a matrix which is highly similar to our $W_h = [X, Y]$ matrix only with displacements instead of positions. They then applied SVD to the matrix, which provided the eigenflows spanning the space of all flow-fields and the coefficients multiplying these basis flow-fields. Temporal cuts were detected at *sign changes of those coefficients*. Their technique can be reformulated in terms of our temporal factorization approach. In our factorization into motion and shape one can view the shape matrix S as being the eigen-vectors spanning the row space and M being the coefficients multiplying these eigen-vectors. Looking at their work this way shows that they

detect cuts at large changes in motion (e.g., shifting from clockwise rotation to counter-clockwise rotation), whereas we detect cuts at non-rigid shape changes and ignore the motion of each shape. Furthermore, reformulating [15] in terms of the temporal factorization approach allows extending it from simple *temporal segmentation* (i.e., detecting cuts) to *temporal clustering*.

Rao & Shah [14] suggested a view-invariant recognition method for complex hand movements. They first obtained hand trajectories (by tracking skin-colored regions) which were sorted according to general structure. Trajectories of similar structure were recognized as the same action by using a low-rank constraint on a matrix constructed from the tracks coordinates. This constraint is equivalent to the one we use for separating between shapes. We detect temporal cuts at increases of the rank and cluster the rows into groups of low rank, i.e., we group frames with the same (or similar) shape.

In a completely different context, Bregler et al. [5] obtained non-rigid object tracking using a factorization/subspace based approach. Their work is *not* related to neither spatial segmentation nor temporal factorization. Nevertheless, we found it appropriate to relate to their work since the shape matrix they used in their decomposition bares similarity to our shape matrix in Eq. (3), which can be misleading. There is a significant difference between their decomposition and ours. They assumed that the shape in each frame is a linear combination of all key-shapes whereas we associate a separate shape with each temporal cluster of frames.

5 Conclusions

We have explored the properties of temporal factorization of the correspondence matrix W and its duality to spatial factorization of the same matrix. We showed that the temporal factorization provides a temporal segmentation and clustering of frames according to non-rigid changes in shape. This approach is unique in the sense that most existing temporal segmentation methods cut the video according to changes in appearance or changes in motion (as opposed to changes in shape).

We showed that to obtain temporal clustering we need *not* develop any new segmentation/clustering scheme but instead can utilize existing algorithms suggested for spatial segmentation. We further showed that dense spatial segmentation requires dense optical flow estimation whereas a small number of tracked points suffices to obtain a dense temporal clustering of frames, i.e., a classification of *all* the frames in the video clip. Furthermore, the dimensionality of the data, which is one of the major difficulties in spatial multi-body factorization, is significantly smaller for temporal segmentation.

The fact that the same factorization framework can be used for spatial segmentation and for temporal segmentation opens new possibilities that may lead to a combined approach for simultaneous spatio-temporal factorization.

6 Acknowledgments

This work was supported by the European Commission (VIBES project IST-2000-26001).

References

1. S. Birchfield. Klt: An implementation of the kanade-lucas-tomasi feature tracker. http://robotics.stanford.edu/~birch/klt/.
2. M. J. Black. Dense optical flow: robust regularization. http://www.cs.brown.edu/people/black/.
3. M. J. Black and P. Anandan. A framework for the robust estimation of optical flow. In *International Conference on Computer Vision*, pages 231–236, Berlin, Germany, 1993.
4. M. J. Black and P. Anandan. The robust estimation of multiple motions: Parametric and piecewise-smooth flow fields. *Computer Vision and Image Understanding*, 63(1):75–104, Jan. 1996.
5. C. Bregler, A. Hertzmann, and H. Biermann. Recovering non-rigid 3d shape from image streams. In *IEEE Conference on Computer Vision and Pattern Recognition*, volume II, pages 690–696, 2000.
6. J. Costeira and T. Kanade. A multi-body factorization method for motion analysis. In *International Conference on Computer Vision*, pages 1071–1076, Cambridge, MA, June 1995.
7. C.W. Gear. Multibody grouping from motion images. *International Journal of Computer Vision*, 2(29):133–150, 1998.
8. M. Irani. Multi-frame correspondence estimation using subspace constraints. *International Journal of Computer Vision*, 48(3):173–194, July 2002.
9. K. Kanatani. Motion segmentation by subspace separation and model selection. In *International Conference on Computer Vision*, volume 1, pages 301–306, Vancouver, Canada, 2001.
10. B.D. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Image Understanding Workshop*, pages 121–130, 1981.
11. M. Machline, L. Zelnik-Manor, and M. Irani. Multi-body segmentation: Revisiting motion consistency. In *Workshop on Vision and Modelling of Dynamic Scenes (With ECCV'02)*, Copenhagen, June 2002.
12. A. Nagasaka and Y. Tanaka. Automatic video indexing and full-video search for object appearances. In *Visual Databas Systems II, IFIP*, 1992.
13. A. Ng, M. Jordan, and Y. Weiss. On spectral clustering: Analysis and an algorithm. In *In Advances in Neural Information Processing Systems 14*, 2001.
14. C. Rao and M. Shah. Motion segmentation by subspace separation and model selection. In *International Conference on Computer Vision*, volume 1, pages 301–306, Vancouver, Canada, 2001.
15. Y. Rui and P. Anandan. Segmenting visual actions based on spatio-temporal motion patterns. In *IEEE Conference on Computer Vision and Pattern Recognition*, June 2000.
16. D.F. Shu S. Swanberg and R. Jain. Knowledge guided parsing in video databases. In *SPIE*, 1993.

17. C. Tomasi and T. Kanade. Shape and motion from image streams under orthography: A factorization method. *International Journal of Computer Vision*, 9:137–154, November 1992.
18. Y. Weiss. Segmentation using eigenvectors: A unifying view. In *International Conference on Computer Vision*, pages 975–982, Corfu, Greece, September 1999.
19. L. Zelnik-Manor and M. Irani. Event-based video analysis. In *IEEE Conference on Computer Vision and Pattern Recognition*, Kauai, Hawaii, 2001.
20. H. Zhang, A. Kankanhali, and W. Smoliar. Automatic partitioning of full-motion video. In *Multimedia Systems*, 1993.

Fig. 4. Temporal vs. spatial clustering using dense optical flow. *Results of factorization applied to a sequence taken from the movie "Brave Heart". The actor (Mel Gibson) is serious at first and then smiles while moving his head independently from his expression throughout the sequence. Optical flow was estimated relative to the first frame and the clustering was applied directly to it. We set the number of clusters to 2 for temporal factorization and to 3 for spatial factorization. (a) Sample frames from the first detected temporal cluster, all of which show the actor smiling. (b) Sample frames from the second detected temporal cluster which show the actor serious. (c) Since optical flow was used, we could obtain dense spatial segmentation. This separated between the forehead, the mouth region and a dangling group of hair. These correspond to three independent motions in the sequence: Along the sequence the actor raises his eyebrows and wrinkles his forehead. Independently of that the mouth region deforms when the actor smiles. The group of hair dingles as the head moves, again independently from the other two motions (the motion of the hair at the lower left part of the image can be seen in the frames in (a) and (b)). Video can be found at http://www.vision.caltech.edu/lihi/Demos/TemporalFactorization.html*