Example Exam

- Answer 4 of the following 5 questions.
- You may write your answers either in Hebrew or in English.
- Length of exam: 3 hours.

Good luck!

Q1:
Explain in short the following terms:
1. Retina.

Q2:
Let \( f(x, y) \) be an \( M \times N \) image, and let \( F(u, v) \) be its Fourier transform. Let \( g(x, y) \) be an image of dimensions \((3M) \times (3N)\), whose Fourier transform \( G(u, v) \) is generated from \( F(u, v) \) by inserting two rows of zeros (0's) between every two rows of \( F \), and two columns of zeros (0's) between every two columns of \( F \):

\[
G(u, v) = \begin{cases} 
F(k, l) & u = 3k, \quad v = 3l \\
& k = 0, \ldots, M - 1 \\
& l = 0, \ldots, N - 1 \\
0 & \text{otherwise}
\end{cases}
\]

What does the image \( g(x, y) \) look like in terms of \( f(x, y) \)? Show mathematically and explain the result in short. Remember that \( g(x, y) \) is of size \((3M) \times (3N)\).

Q3:
Background in inpainting:
Let \( x \) be an image with \( N \) pixels and suppose that the values of \( M \) pixels are missing. We have seen in class that this can be expressed as an \((N - M)\) element signal \( y \) such that \( y = Ax \), where \( A \) is an \((N - M) \times N\) matrix. The rows of \( A \) are a subset of rows from an \( N \times N \) identity matrix, where all the rows representing missing pixels have been removed (in Matlab notation, let \( E = \text{eye}(N) \), \( A = E(\text{inds},:) \), where \( \text{inds} \) is the list of \( N - M \) observed entries). We assume a smoothness prior on the derivatives of \( x \)

\[
p(x) = e^{-\frac{1}{\sigma^2} \sum_{i,j} |g_{ij}^0(x)|^\alpha + |g_{ij}^1(x)|^\alpha}
\]  

(1)
where \( g_{ij}^x(x) \) denotes the horizontal derivative of the image \( x \) at pixel \((i,j)\), and \( g_{ij}^y(x) \) denotes the vertical derivative of the image \( x \) at pixel \((i,j)\).

To recover image \( x \) from image \( y \) we solve the following minimization problem:

\[
x^* = \arg \min_x \frac{1}{\eta^2} \|Ax - y\|^2 + \frac{1}{\sigma^2} \sum_{i,j} |g_{ij}^x(x)|^\alpha + |g_{ij}^y(x)|^\alpha
\]  

Answer the following questions:

1. How would you minimize Eq 2 if a Gaussian prior \((\alpha = 2)\) is used?
2. How would you minimize Eq 2 if a Sparse prior \((\alpha < 1)\) is used?

Q4:

Let \( P_i = (X_i, Y_i, Z_i)^T \), \( i = 1, ..., N \) be a set of 3D points in space with image projections \( p_i = (x_i, y_i, f)^T = \frac{f}{Z_i} P_i \), where \( f \) is the focal length of the camera. The camera undergoes a rigid motion,

\[
P_i' = RP_i + t
\]

where \( R \) is the rotational component and \( t \) is the translational component of camera motion. The projections in the new view are \( p_i' = \frac{f'}{Z_i'} P_i' \), where \( f' \) is the focal length in the new view. All the \( N \) points are visible in both images, and the correspondences \( p_i \leftrightarrow p_i' \) are given.

Is it possible to recover the depth \( Z_i \) of all/any/none of the points in each of the following cases:

(a) Pure camera rotation.
(b) Pure camera zoom \((f' \neq f)\).
(c) Pure camera translation.

Show why.

Q5:

A camera is imaging an object at two time instances. Point correspondences are given across the two images. Can depth be recovered if:

1. There is only a camera rotation between the two images?
2. There is only an object rotation between the two images?