Introduction to Computer Vision, Winter 2011

Final Exam

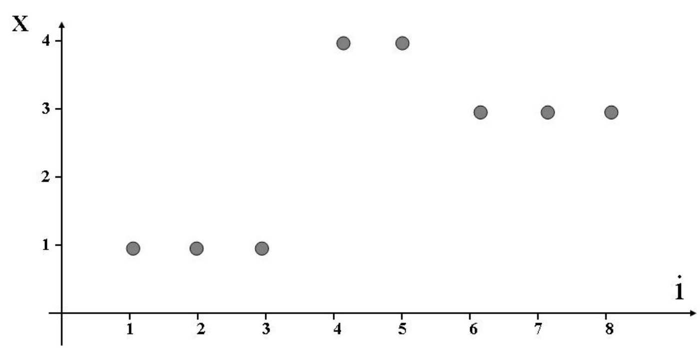
- Answer 4 of the following 5 questions.
- You may write your answers either in Hebrew or in English.
- Length of exam: 3 hours.

Good luck!

Q1:

1. Is it possible to discretely sample a function $f : \Re \longrightarrow \Re$ at uniform finite distances without loosing any information? (i.e., can we sample $f(x)$ discretely, and be able to reconstruct the complete $f(x)$ back from its discrete samples?) Explain when.
2. Let $f(x)$ be a function that can be recovered from discrete uniformly spaced samples, and let $g(x)$ be a function that can NOT be recovered from any such discrete sampling. For each of the following functions state whether or not it is recoverable from some discrete uniformly spaced sampling. Explain why.
   (a) $h_1(x) = f(x) + g(x)$.
   (b) $h_2(x) = f(x) * g(x)$ ('$*$' denotes convolution).
3. What does the following function look like? $f_3(x) = sin(x) * cos(2x)$. Explain why.

Q2: We are given the discrete 8 element signal (denoted x_i) plotted below:



We define affinities between every pair of neighboring pixels as:

$$w_{i,i+1} = \begin{cases} 2 & \quad\quad |x_i - x_{i+1}| < 1 \\ 1 & 1 \leq |x_i - x_{i+1}| < 2 \\ 0.5 & 2 \leq |x_i - x_{i+1}| \end{cases} \tag{1}$$

1. We want to segment this signal into two groups under the constraint that $i = 2$ must belong to one of the groups and $i = 6$ to the other. For that we wish to assign each pixel a label $\ell(i) \in \{\alpha, \beta\}$ minimizing neighbors dissimilarity:

$$E\left(\ell(1), ..\ell(8)\right) = \sum_i w_{i,i+1}[\ell(i) \neq \ell(i+1)] + \lambda \cdot \left([\ell(2) \neq \alpha] + [\ell(6) \neq \beta]\right)$$

   where $\lambda \to \infty$ is a large number.

   (a) Explain how to cast this problem into a min-cut problem in a graph.

   (b) Draw the graph you would construct.

   (c) What would the optimal assignment be?

2. We now want to segment the signal into 3 groups, under the following constraints $\ell(2) = \alpha, \ell(6) = \beta, \ell(8) = \gamma$. We start with the following label assignment:

$$\ell(1) = \gamma,\ \ell(2) = \alpha,\ \ell(3) = \alpha,\ \ell(4) = \gamma,\ \ell(5) = \gamma,\ \ell(6) = \beta,\ \ell(7) = \alpha,\ \ell(8) = \gamma$$

   Draw the graphs corresponding to

   (a) $\alpha - \gamma$ swap step

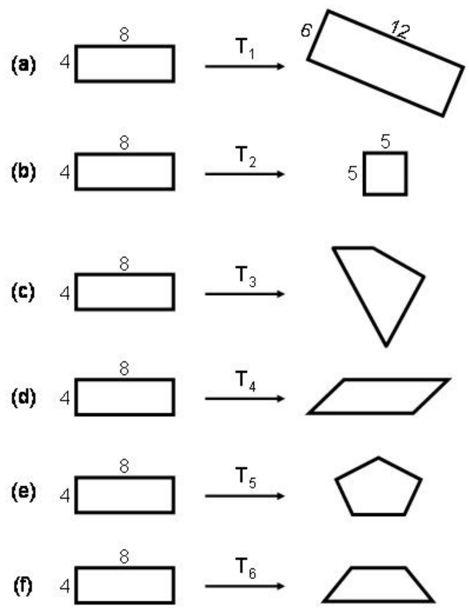
   (b) $\beta - \gamma$ swap, applied to the labeling result of step (a).

   What would the labels be after each step?

Q3:

1. Write an expression for the following transformations and state what the minimal number of point correspondence required for recovering each transformation:
   (a) 2D rigid.
   (b) 2D similarity.
   (c) 2D affine.
   (d) 2D projective.

2. For each of the transformations above state whether it can be used to model the transformation T_i in each of the drawings below.



Q4: Two **stationary** video cameras record a billiard table (the top flat part only).

(a) What is the geometric relation between the images obtained by the two cameras?

(b) At any one time the cameras see exactly two ball, a red and a green ball. (Neither the table boundaries, nor any markings, nor any of the other balls are in the fields of view of the two cameras). The red ball is moving, while the green ball is stationary. What is the minimal number of frames required to compute the geometric relation between the images obtained by the two cameras? (i.e., the geometric relation in (a)).

(c) Is this minimal number of frames always sufficient? (e.g., could there be types of ball motions for which this number is not enough?) Explain.

Q5:

(a) Explain the concepts of: feature space, linear separator, SVM.

(b) Give two examples of features that could be used to separate between two visual classes.

(c) Write an expression of what needs to be optimized to find the SVM classifier from examples.