## Appendix– Deriving the SR-kernel for SR×4 from the SR-kernel for SR×2

We will show that the kernel for a scale factor of 4, i.e. $k_4$, can be analytically calculated from $k_2$.

**Claim**:
Given a kernel $k_2$, let $k_4$ be defined by:

$I_{LR} * k_4 \downarrow_4 = (I_{LR} * k_2 \downarrow_2) * k_2 \downarrow_2$.

Then, $k_4 = k_2 \ * \ k_{2\_dilated}$ , where $k_{2\_dilated}[n_1, n_2] = \begin{cases} k_2\left[\frac{n_1}{2}, \frac{n_2}{2}\right] & n_1, n_2 \text{ even} \\ 0 & \text{else} \end{cases}$

**Proof**:
For simplicity, we will assume 1D signals, but this can be easily generalized to 2D.

Let us define: $x := I_{LR} * k_2$.

$$\left\{(I_{LR} * k_2 \downarrow_2) * k_2\right\}[n] = \left\{k_2 \ * \ x[2\,\cdot\,]\right\}[n] = \sum_{l=-\infty}^{\infty}\left(k_2[n-l] \cdot x[2l]\right)$$

We can substitute $m := 2l$ and use Kronecker Delta Comb to enforce $m$ to be even;

$$= \sum_{l=-\infty}^{\infty}\left(\left(\sum_{m=-\infty}^{\infty}\delta[2l-m]\right) \cdot k_2[n-\frac{m}{2}] \cdot x[m]\right)$$

Assuming both signals are of finite energy, changing the summation order is allowed. Adding $2n$ does not change the result of the Kronecker Delta Comb;

$$= \sum_{m=-\infty}^{\infty}\left(\left(\sum_{l=-\infty}^{\infty}\delta[2l-m+2n] \cdot k_2\left[\frac{1}{2}(2n-m)\right]\right) \cdot x[m]\right)$$

This formulates a convolution;

$$= \left\{\left(\sum_{l=-\infty}^{\infty}\delta[\,\cdot\, + 2l] \cdot k_2\left[\frac{\cdot}{2}\right]\right) \ * \ x\right\}[2n]$$

The Kronecker Delta Comb zeroizes for odd inputs, we therefore obtain $k_{2\_dilated}$;

$$= \left\{k_{2\_dilated} \ * \ x\right\}[2n]$$

Plug in $x := I_{LR} * k_2$ and subsample both sides by $\downarrow_2$

$(I_{LR} * k_2 \downarrow_2) * k_2 \downarrow_2 = I_{LR} * (k_2 \ * \ k_{2\_dilated}) \downarrow_4$ □