# A Sub-Nyquist Radar Demo System: Hardware and Software

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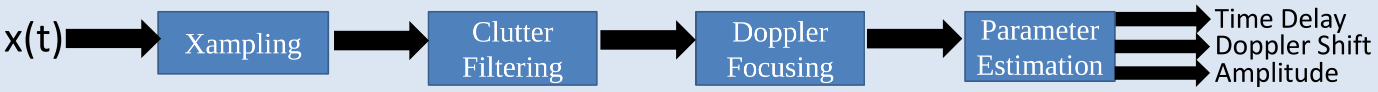
## Motivation and Goals

- ❑ High resolution radar requires high bandwidth signals
- ❑ Wideband signals need a complex analog front end receiver design which consumes high power
- ❑ Digital processing of wideband signals requires large memory and large computational power
- ❑ We present a sub-Nyquist sampling and recovery method implemented in hardware which reduces the rate by 30 fold
- ❑ This approach provides both simple recovery and robustness to noise by performing beamforming on the low rate samples
- ❑ Clutter rejection is also performed on the sub-Nyquist samples by adapting standard methods to our setting

## Sub-Nyquist Radar Algorithm



x(t) → Xampling → Clutter Filtering → Doppler Focusing → Parameter Estimation → Time Delay, Doppler Shift, Amplitude

- ❑Xampling– A process of sampling a signal at a low rate in such a way that preserves the required information
- ❑Clutter Filtering – Adaptation of standard clutter algorithms to fit our low rate samples
- ❑Doppler Focusing - A method of digitally beamforming the low rate samples which is both numerically efficient and robust to noise
- ❑Estimation – A modified OMP, matched to our samples, produces target locations and Doppler frequencies

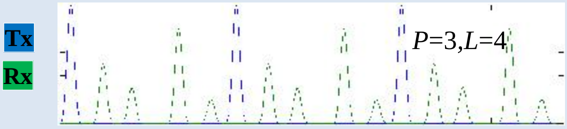
## Input Signal Model

- ❑$L$ targets, each defined by 3 degrees of freedom: amplitude $\alpha_\ell$, delay $\tau_\ell$, and Doppler frequency $\nu_\ell$
- ❑After transmitting $P$ equispaced high-bandwidth pulses $h(t)$, the received signal*:

$$x(t)=\sum_{p=0}^{P-1}\sum_{l=0}^{L-1}\alpha_l h\left(t-\tau_l-p\tau\right)e^{-j\nu_l p\tau}$$

(* some assumptions on target dynamics are needed for this model)

- ❑This is an FRI model as $x(t)$ is completely defined by $3L$ parameters



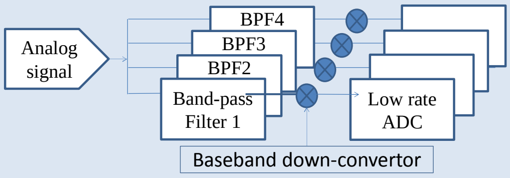
- ❑The signal's Fourier coefficients contain the required parameters:

$$\boxed{c_p[k]=\frac{1}{\tau}\int_0^{\tau}x_p(t)e^{-j2\pi kt/\tau}dt=\frac{1}{\tau}H(2\pi k/\tau)\sum_{\ell=0}^{L-1}\alpha_\ell e^{-j2\pi k\tau_\ell/\tau}e^{-j\nu_\ell p\tau}}$$

- ❑Standard radar methods sample and process at the Nyquist rate

## Xampling Scheme – Acquiring Fourier Coefficients

- ❑The signal's parameters are embodied in its Fourier coefficients
- ❑Multichannel analog processing and low rate sampling scheme are used to extract spectral information for specific frequency bands:



- ❑Calculating Fourier coefficients is performed digitally after sampling

## Clutter Filtering

- ❑ The target signal is contaminated with clutter + thermal noise:

$$y(t)=x(t)+c(t)+n(t)$$

- ❑ Assume the clutter interference is modelled as "colored" noise - a WSS random process whose spectrum is Gaussian:

$$S_C(f)=P_C\cdot\frac{1}{\sqrt{2\pi}\sigma_c}\cdot exp\left[\frac{(f-f_c)^2}{2\sigma_c^2}\right]$$

- ❑ Clutter + Thermal Noise autocorrelation matrix:

$$M(m,n)=(P_c/P_N)e^{-2\left(\pi\sigma_c T\cdot(m-n)\right)^2}e^{-j\cdot 2\pi(m-n\,)f_c}+\delta_{m,n}$$

- ❑ Filtering is performed by using the whitening matrix $M^{-\frac{1}{2}}$ to whiten the interference and proceeding with Doppler focusing.

Sampled Fourier Coefficients $c[k]$ → Whitening → Doppler Focusing

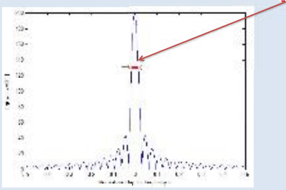
## Doppler Focusing

- ❑Transforms a simultaneous Delay-Doppler estimation problem into a set of delay-only problems with specific Doppler frequency
- ❑Focusing on Doppler frequency $v$ for sampled Fourier coefficients:

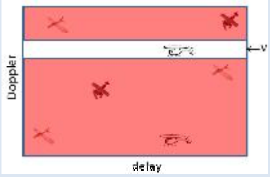
$$\boxed{\Psi_\nu[k]=\sum_{p=0}^{P-1}\tilde{C}_p\,[k]e^{j\nu p\tau}=\frac{1}{\tau}\underbrace{H(2\pi k/\tau)\sum_{\ell=0}^{L-1}\alpha_\ell e^{-j2\pi k\tau_\ell/\tau}}_{\text{Spectral analysis problem}}\underbrace{\sum_{p=0}^{P-1}e^{j(\nu-\nu_\ell)p\tau}}_{\text{Focusing term}}}$$

Focusing term

$$\sum_{p=0}^{P-1}e^{j(\nu-\nu_\ell)p\tau}\cong\begin{cases}P, |\nu-\nu_\ell|<\pi/P\tau\\0, \quad otherwise\end{cases}$$

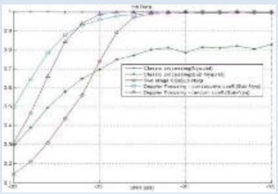


- ❑Coherent integration of echoes from different pulses creates a single superimposed pulse. SNR scaling is linear with $P$
- ❑Instead of detecting targets in the delay- Doppler plane, Doppler focusing creates slices in which targets are detected using delay only
- ❑**A hard 2D estimation problem is efficiently reduced into several easier 1D problems.**

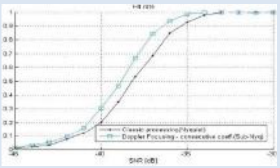


## Simulation Results

- ❑Measure performance by"hits" and RMS error
- ❑ A "hit" is a Delay-Doppler estimate in the interior of an ellipse around the true target one tenth the Nyquist Rate and at -25bB SNR, Doppler focusing achieves performance equivalent to matched filter processing sampling at the Nyquist rate



- ❑ When we concentrate the signal's energy contents in the sampled frequencies, Doppler focusing outperforms matched filtering at Nyquist rate



- ❑ Under SNR of -16dB and 100 pulses used:
- ❑ Without clutter filtering, only 3 out of 5 targets are detected
- ❑ Using clutter filtering algorithm, all 5 targets are detected.
- ❑ **The performance of sub-Nyquist algorithm is equivalent to classic Nyquist rate processing.**