CORRECTION SHEET
AN INTRODUCTION TO RANDOM MATRICES
Anderson, Guionnet, Zeitouni
Date: September 4, 2018

This list collects the most current list of corrections for the above book. Items marked by * are important, that is, are more than typos. We thank the following people for their comments: Florent Benaych-Georges, Jun Chen, Amir Dembo, Svante Janson, Toby Johnson, Achim Klenke, Mylène Maïda, Edouard Maurel Segala, Alain Rouault.

(1) Page 10, line 2, replace 2 in the denominator by 2.
(2) * Page 15, line 22 and 30, replace “the next-to-last letter of \( w_{i-1} \)” by “the entry preceding the first occurrence of the last letter of \( w_{i-1} \).”
(3) * Page 13, line 23, replace “\( \ell(\text{w})-1 \) edges” by “at most \( \ell(\text{w})-1 \) edges”.
(4) Page 23, line 3, add a missing | after \( f(y) \).
(5) Page 32, line 4, replace “fail” by “fails”.
(6) Page 36, line -3, replace \( T_{Ni} \) by \( \bar{T}_{Ni} \).
(7) Page 37, (2.2.4), replace “\( g(1,1) \)” by “\( g(1,1) \)k/2”.
(8) Page 40, line -2, replace “\( G \wedge (-1/\epsilon) \lor (1/\epsilon) \)” by “\( G \wedge (1/\epsilon) \lor (-1/\epsilon) \)”.
(9) Page 41, line 3, replace “= \( M \ldots \)” by “\( \leq \sqrt{M} \ldots \).” In (2.3.8), replace \( E \) in left side by \( E_p \).
(10) Page 49, Equation (2.4.14), replace \( S_\mu(\lambda+\epsilon) \) by \( S_\mu(\lambda+i\epsilon) \).
(11) Page 59, line -4, replace \( \Delta(x)^{2\epsilon} \) by \( |\Delta(x)|^{2\epsilon} \).
(12) Page 73, in (2.6.10), replace “for all” by “for (Lebesgue) almost all”. (In fact, the set where the strict inequality does not hold has vanishing logarithmic capacity.)
(13) Page 76, line -7, Page 77, lines 6 and 17, replace \( Z_N^{\beta,V} \) by \( Z_N^{V,\beta} \).
(14) Page 78 line -4, -2, -1, page 79 line 3, replace \( \nu \) by \( \mu \).
(15) Page 79 line 7, replace \( \epsilon \) by \( \delta \).
(16) Page 79, line -6, display, replace “\( \lim_{N \to \infty} \)” by “\( \lim_{\delta \to 0} \)”.
(17) Page 79 line -4, replace \( \lambda_i < \lambda_{i-1} \) by \( \lambda_i < \lambda_{i+1} \).
(18) Page 80, lines 14, 17, replace \( Z_N^{\beta,V} \) by \( Z_N^{V,\beta} \).
(19) Page 81, line 7, replace (2.4.6) by (2.4.7).
(20) * Page 81, Assumption 2.6.5: add the assumption that either \( J^{\beta,V}(\cdot) \) achieves its minimum at \( x^* \) only, or that under \( P_{NV/(N-1),\beta}^{N-1} \), the top eigenvalue converges in probability to \( x^* \). In the former case, one need to use Jensen’s inequality in order to show that \( J^{\beta}(\cdot) \geq 0 \).
(21) * Page 81, display in Theorem 2.6.6 has a sign error, and should read
\[
J^{\beta}(x) = \begin{cases}
-\beta \int \log |x-y|\sigma^{V}_\beta(dy) + V(x) - \alpha_{V,\beta} & \text{if } x \geq x^*, \\
\infty & \text{otherwise},
\end{cases}
\]
(22) Page 91, line 6, replace \( \sqrt{n} \) by \( \sqrt{N} \).
(23) Page 93, line -4, replace Corollary 3.1.5 by Theorem 3.1.5.
By applying column operations we have det Ψ_A(z) = det Θ_A(z). Define

\( \hat{\Theta}_A(z) = \left[ \begin{array}{c} \epsilon(1_A f_i)(z_j) \\ f_i(z_j) - \epsilon(1_A f_i)(z_r) \end{array} \right]_{n,1} \)

and the “deformed matrix”

\[ \hat{\Psi}_A(z) := (\Theta_A + \hat{\Theta}_A)(z) = \left[ \begin{array}{c} \epsilon(1_A f_i)(z_j) \\ f_i(z_j) - \epsilon(1_A f_i)(z_r) \end{array} \right]_{n,1} \] .

Applying obvious column operations one gets det \( \hat{\Psi}_A(z) = c_1 \) det \( \hat{\Theta}_A(z) \), for a nonzero constant \( c_1 \) independent of \( A \) and \( z \). Finally, one has

\[
\int_{A^\cap[z_{r-1},t]} (\hat{\Psi}_A(z_1, \ldots, z_{r-1}, t) - \hat{\Psi}_A(z_1, \ldots, z_t, t)) dt
\]

\[
= \int_{A^\cap[z_{r-1},t]} \hat{\Theta}_A(z_1, \ldots, z_{r-1}, t) dt
\]

\[
= \det \left[ \begin{array}{c} \epsilon(1_A f_i)(z_j) \\ f_i(z_j) - \epsilon(1_A f_i)(z_r) \end{array} \right]_{n,1} = 0
\]

Integrating over the remaining coordinates yields (0.1).
(35) Page 190, line 18, display: replace $e^{-\beta x_i/4}$ by $e^{-\beta x_i/2}$ (recall that standard Normal is defined in pages 188-189).

(36) Page 208, line 4, replace $\text{Mat}_{p\times q}$ by $\text{Mat}_{p\times(q-p)}$.

(37) * Page 208, Proposition 4.1.33: Unfortunately, Assumption (IId) in the definition of the Weyl quadruple is not satisfied in the setup described in Proposition 4.1.33. The following correction shows that the conclusion of the proposition still holds.

The only use of Assumption (IId) is in the proof of

$$T_{I_n}(f_{\lambda})(T_{I_n}(G)) \subset T_{\lambda}(M) \cap T_{\lambda}(\Lambda)^\perp$$

(4.1.24)

in Lemma 4.1.26, see (4.1.27). Since (4.1.23) already shows that $T_{I_n}(f_{\lambda})(X) = [X, \lambda]$, and since the inclusion in $T_{\lambda}(M)$ is automatic, it is enough to verify that

$$[X, \lambda] \cdot \tau = 0 \text{ for } X \in T_{I_n}(G), \lambda \in \Lambda \text{ and } \tau \in T_{\lambda}(\Lambda).$$

(0.2)

In particular, (0.2) should replace Assumption (IId) in the definition of the Weyl quadruple.

We now check (0.2) under the assumptions of Proposition (4.1.33). Recall that $0 < p \leq q$, $n = p + q$, $0 \leq r \leq q - p$ and $q = p + r + s$. Fix a point

$$\lambda = \text{diag}(x, y, I_p - x, I_r, 0_s) \in \Lambda.$$

Recall that $x^2 + y^2 = x$. By implicit differentiation of the last matrix equation we deduce that

$$T_{\lambda}(\Lambda) = \{\text{diag}(\left[\begin{array}{cc} \xi & \eta \\ \eta & -\xi \end{array}\right], 0_{r+s}) \mid \xi, \eta \in \text{Mat}_p \text{ are diagonal and } (2x - I_p)\xi + 2y\eta = 0\}.$$

Fix a tangent vector

$$\tau = \text{diag}(\left[\begin{array}{cc} \xi & \eta \\ \eta & -\xi \end{array}\right], 0_{r+s}) \in T_{\lambda}(\Lambda).$$

We have

$$T_{I_n}(G) = \{\text{diag}(P, Q) \mid P \in T_{I_p}(U_p(F)), Q \in T_{I_q}(U_q(F))\}.$$

We arbitrarily fix

$$p \quad p \quad r \quad s$$

$$\begin{array}{cccc}
p & & & \\
p & a & 0 & 0 \\
r & 0 & b & * \\
s & 0 & * & * \\
s & 0 & * & * \\
\end{array} \in T_{I_n}(G)$$
where we have broken down into blocks of the indicated sizes, and the blocks marked by * will be irrelevant to the computation, and therefore we do not specify their values. We get

\[
\begin{bmatrix}
  a & 0 & 0 & 0 \\
  0 & b & * & * \\
  0 & * & * & * \\
  0 & * & * & * \\
\end{bmatrix}
\begin{bmatrix}
  x & y & 0 & 0 \\
  y & I_p - x & 0 & 0 \\
  0 & 0 & I_r & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
  ax & ay & 0 & 0 \\
  by & b(I_p - x) & * & * \\
  * & * & * & 0 \\
  * & * & * & 0 \\
\end{bmatrix}.
\]

and therefore

\[
\begin{bmatrix}
  x & y & 0 & 0 \\
  y & I_p - x & 0 & 0 \\
  0 & 0 & I_r & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  a & 0 & 0 & 0 \\
  0 & b & * & * \\
  0 & * & * & * \\
  0 & * & * & * \\
\end{bmatrix}
= 
\begin{bmatrix}
  xa & yb & * & * \\
  ya & (I_p - x)b & * & * \\
  0 & * & * & * \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}.
\]

In turn, by exploiting all the zeroes above, we get \([X, \lambda] \cdot \tau = [a, x] \cdot \xi + [b, x] \cdot \xi + (ay - yb) \cdot \eta + (by - ya) \cdot \eta.\) Taking into account that \(x, y, \xi, \eta\) are real and diagonal, it implies that indeed \([X, \lambda] \cdot \tau = 0\) as required by (0.2).

(38) Page 214, second display, the expressions in the definitions of \(D(\alpha, \beta)\) and \(C(\alpha)\) should be squared.

(39) * Page 219, equation (4.2.8), replace \(\prod_{i=1}^{k} D_i \times D^r\) by \(\prod_{i=1}^{k} D_i \times (D \setminus \cup_{i=1}^{k} D_i)^r).\)

(40) * Page 244, display (4.2.51): add the assumption that \(\mu^N\) is a positive measure of total mass \(N!\).

(41) Page 284, 3 lines above Corollary 4.4.4, replace \(a^2 m^{-1}\) by \(a^{-2} m\).

(42) Page 286, Line -7, replace \(\eta_j\) by \(\zeta_j\).


(44) Page 429, line 4 of Lemma D.9, replace \(A\) by \(B\).