Errata for the paper: *Quenched, annealed and functional large deviations for one-dimensional random walk in random environment*, by F. Comets, N. Gantert, O. Zeitouni.
Prob. Theory Rel. Fields 118 (2000), pp. 65–114.

1. The argument for the convexity of I_η^q at 0 given at the last two lines of page 92 is wrong. It is correct that I_η^q is convex on $(0,1]$ and on $[-1,0)$, separately. To show convexity, it is enough to prove that $(I_\eta^q)'(0^-) \le (I_\eta^q)'(0^+)$. But

$$(I_\eta^q)'(0^-) + (I_\eta^q)'(0^+) = \int \log \rho_0(\omega)\eta(d\omega) \tag{$*$}$$

and not as written. A correct proof of the convexity proceeds as follows: Using $(*)$, we see that it suffices to show that

$$(I_\eta^q)'(0^+) \ge \frac{1}{2}\int \log \rho_0(\omega)\eta(d\omega)\,. \tag{$**$}$$

Due to (5), (6) and Lemma 2,

$$(I_\eta^q)'(0^+) = -\int \log E_\omega[e^{\lambda_{\text{crit}}\tau_1}]\eta(d\omega) = -\int \log \varphi(\lambda_{\text{crit}},\omega)\eta(d\omega)\,.$$

By (34) and monotone convergence, for $\lambda \le \lambda_{\text{crit}}$,

$$\varphi(\lambda,\omega)\tilde\varphi(\lambda,\theta^{-1}\omega) \le 1\,.$$

Taking logarithms and integrating with η, we conclude that

$$\int \log \varphi(\lambda,\omega)\eta(d\omega) + \int \log \tilde\varphi(\lambda,\omega)\eta(d\omega) \le 0.$$

Combined with Lemma 3, we deduce that for all $\lambda \le \lambda_{\text{crit}}$, $2\int \log\varphi(\lambda_{\text{crit}},\omega)\eta(d\omega) \le -\int \log\rho_0(\omega)\eta(d\omega)$, hence $(**)$ is satisfied.

2. One has to add a missing term in the statement of Lemma 2, part (ii). It should read:
Let $u_{\text{crit}} = \infty$ *if*

$$\int \frac{E_\omega[\tau_1 e^{\lambda_{\text{crit}}\tau_1}]}{E_\omega[e^{\lambda_{\text{crit}}\tau_1}]}\eta(d\omega) = \infty$$

and .....
The proof given relates to this statement.

3. In formulas (70) and (73), the sup should be replaced by a summation. Further, the proof of (73) needs an additional ingredient due to the appearance of shifts in the right hand side, namely one first checks that the logarithmic limit of the right hand side of (73) is estimated in probability, and hence a.s. along subsequences, by $-I_\eta^q(v)$, and then one uses that the left hand side of (74) is constant P-a.s.

4. Finally, a clarification is needed concerning the use of the Minimax theorem in the proof of the annealed upper tail, top of page 90: indeed, to apply the Minimax theorem, first note that the last expression in (63) equals

$$-\sup_{0\le\lambda\le\lambda_{\text{crit}}}\ \inf_{\eta\in M_1^s(\Sigma)^K} [G(\lambda,\eta,u) + h(\eta|\alpha)]$$

where $\lambda_{\text{crit}} = \lambda_{\text{crit}}(\alpha)$. Due to the Minimax theorem, the last expression equals

$$-\inf_{\eta\in M_1^s(\Sigma)^K}\ \sup_{0\le\lambda\le\lambda_{\text{crit}}} [G(\lambda,\eta,u) + h(\eta|\alpha)]$$

Exactly as we proved (57), one can pass from stationary to ergodic measures and show

$$-\inf_{\eta\in M_1^s(\Sigma)^K}\ \sup_{0\le\lambda\le\lambda_{\text{crit}}} [G(\lambda,\eta,u) + h(\eta|\alpha)] = -\inf_{\eta\in M_1^e(\Sigma)^K}\ \sup_{0\le\lambda\le\lambda_{\text{crit}}} [G(\lambda,\eta,u) + h(\eta|\alpha)] \tag{$***$}$$

instead of (64). The problem in passing to (65) is that, if $h(\eta|\alpha) < \infty$, $\lambda_{\rm crit}(\eta) \geq \lambda_{\rm crit}(\alpha)$ but one does not have equality in general. Therefore, the relation to the variational problem involving $I_\eta^{\tau,q}(\cdot)$ is not immediate. To bypass this obstacle, note that, first by replacing η with $(1-n^{-1})\eta + n^{-1}\alpha$ and then approximating with a sequence of ergodic measures, the last expression in $(***)$ equals

$$-\inf_{\{\eta\in M_1^e(\Sigma)^K, \lambda_{\rm crit}(\eta)=\lambda_{\rm crit}\}} \sup_{0\leq\lambda\leq\lambda_{\rm crit}} \left[G(\lambda,\eta,u) + h(\eta|\alpha)\right] .$$

From here, one proceeds as in the case of the lower tail, concluding that

$$\begin{aligned}
&\limsup_{n\to\infty} \frac{1}{n} \log P\left[\infty > T_n/n \geq u\right] \\
\leq\ & -\inf_{\{\eta\in M_1^e(\Sigma)^K, \lambda_{\rm crit}(\eta)=\lambda_{\rm crit}\}} \sup_{0\leq\lambda\leq\lambda_{\rm crit}} \left[G(\lambda,\eta,u) + h(\eta|\alpha)\right] \\
=\ & -\inf_{\{\eta\in M_1^e(\Sigma)^K, \lambda_{\rm crit}(\eta)=\lambda_{\rm crit}\}} \inf_{w\geq u} I_\eta^{\tau,q}(w) \leq -\inf_{\eta\in M_1^e(\Sigma)^K} \inf_{w\geq u} I_\eta^{\tau,q}(w) = -\inf_{w\geq u} I_\alpha^{\tau,a}(w)\,,
\end{aligned}$$

where the first equality is due to (41), yielding (66).