Control and Optimal Control

Assignment 1, due November 30, 2011

1. Consider two mass springs tied together with one tied to a wall. Write the differential system that governs the coupled springs, when a control force can be applied. Consider three cases: One when forces can be applied independently to the two mass; the second when only the inner mass is subject to the force; the third when the outer mass is subject to the force.

2. In the previous exercise, in which of the three cases is the system controllable?

3. Consider 3 compartments arranged in a circle, with volumes V_1, V_2, V_3, each containing a well mixed solution, with sides well insulated except the ones between any two of them. The diffusion between any two is subject to a diffusion coefficient $k_{i,j}$ with $i \neq j$ each between 1 and 3. Substance can be added to compartment no. 1. Write the differential equation governing the concentration (x_1, x_2, x_3) of solution in the three compartments.

4. In the previous exercise, under what conditions on the V_i and the $k_{i,j}$ are the concentrations controllable?

5. Prove that if A and B are constant and if $\mathrm{Im}B$ has rank k then controllability holds if and only if the matrix $[B, AB, \ldots, A^{n-k}B]$ has rank n

6. Prove that in the time invariant case a state not in the controllable space cannot be steered to a point in the controllable space. (The controllable space is the set of points to which 0 can be steered.)

7. Show that the formula
$$x(t) = e^{\int_{t_0}^t A(\tau)d\tau} x_0$$
may not constitute a solution to $\frac{dx}{dt} = A(t)x$