

## Control and Optimal Control

### Assignment 2, due December 7, 2011

1. Find for what  $\Delta$  the harmonic oscillator  $\frac{d^2x}{dt^2} + x = u$  is controllable when controls must be constant on intervals of Length  $\Delta$ ?
2. Prove that the system

$$\begin{pmatrix} \frac{d\xi_1}{dt} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \frac{d\xi_n}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 \\ a_0 & \cdot & \cdot & \cdot & \cdot & a_{n-2} & a_{n-1} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \xi_n \end{pmatrix} + \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b \end{pmatrix} u$$

is controllable.

3. A *linear state feedback* is a mapping  $u = Kx$  with  $K$  a matrix. Prove that if  $(A, B)$  is controllable then  $(A + BK, B)$  is also controllable. What could be the interpretation, possibly the advantage, of replacing  $A$  with  $A + BK$ ?
4. Give an example of a time invariant linear system that is not controllable.