Control and Optimal Control

Assignment 3, due December 14, 2011

1. Explain why a system
$$\frac{dx}{dt} = A(t)x + B(t)u \tag{1}$$
that is controllable on and interval $[t_0, t_1]$ is also controllable on any interval that contains $[t_0, t_1]$.

2. Show that a system of the type (1) may not be controllable on two intervals $[t_0, t_1]$ and $[t_1, t_2]$ yet controllable on $[t_0, t_2]$.

3. Show that if (1) is controllable on $[t_0, t_1]$ then it is controllable on $[t_0 + \varepsilon, t_1 - \varepsilon]$ for ε small enough.

4. True or false: If for every fixed $t \in [t_0, t_1]$ the coefficients $(A(t), B(t))$ form a controllable pair (as a time invariant system) then (1) is controllable on $[t_0, t_1]$.

5. Is the system
$$\begin{pmatrix} \frac{d\xi_1}{dt} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \frac{d\xi_n}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 \\ a_0(t) & \cdot & \cdot & \cdot & \cdot & a_{n-2}(t) & a_{n-1}(t) \end{pmatrix} \begin{pmatrix} \xi_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \xi_n \end{pmatrix} + \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b(t) \end{pmatrix} u$$
controllable in an interval where if $b(t) > 0$ on that interval? (Hint: no need to resort to the characterization of controllaniity via the controllability matrix).