Control and Optimal Control

Assignment 4, due December 21, 2011

Consider the problem of driving a state x to zero along an interval $[0, \tau]$ with a control system

$$\frac{dx}{dt} = Ax + Bu$$

(i.e. time invariant) while minimizing the cost

$$\int_o^\tau |u(t)|^2 dt.$$

Assume the system is controllable.

1. Show (via direct calculation employing the solution given at class) that the optimal cost is a quadratic function of x.
2. Show that the cost is non-increasing in τ hence converges as $\tau \to \infty$.
3. Show that the cost converges to ∞ as $\tau \to 0$
4. Show that the cost converges to ∞ as $|x| \to \infty$
5. Which of the former 4 statements stays correct when the system is time varying (i.e. $A = A(t)$ and $B = B(t)$ depend on time), assuming that the system is controllable on any interval.