Control and Optimal Control

Assignment 5, due January 4, 2012

1. Consider 3 compartments arranged in a circle, with volumes V_1, V_2, V_3, each containing a well mixed solution, with sides well insulated except the ones between any two of them. The diffusion between any two is subject to a diffusion coefficient $k_{i,j}$ with $i \neq j$ each between 1 and 3. The content of compartment no. 1 can be measured. Under what conditions on the V_i and the $k_{i,j}$ is the system observable?
2. For a set E in R^n and a vector p in R^n we write $p(E) = \max\{px : x \in E\}$. Prove that the Hausdorff distance between two sets C and D is equal to the maximum among $p \in R^n,\ |p| = 1$ of $|p(C) - p(D)|$.
3. Let $R(t)$ be a compact set that depends continuously (with respect to the Hausdorff distance) on the time t, where $t \in [t_0, t_1]$. Suppose that a point $x \in R^n$ belongs to $R(t)$ for some t. Prove that the set of t for which $x \in R(t)$ is closed. In particular a minimal time, say τ, with this property exists.
4. Show that in the previous exercise, if $R(t)$ is convex for every t, for the minimal time τ the point x is a boundary point of $R(\tau)$.
5. Show that the convexity cannot be dropped in the previous exercise.