Control and Optimal Control

Assignment 6, due January 18, 2012

1. Consider the double integrator
$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = u$$
with control constraint by $|u| \leq 1$. Show that any point in R^2 can be steered to any other point. Prove that the minimal steering time is achieved with bang-bang control that switched only once between 1 and -1.

2. Consider the harmonic oscillator with friction
$$\begin{aligned} \frac{dx}{dt} &= \ y \\ \frac{dy}{dt} &= \ -x - y + u \end{aligned}$$
with control constraint by $|u| \leq 1$. Explain why there are points in R^2 to which a prescribed point, say 0 cannot be steered at all. What is the form of the control function that steers one point to another in minimal time? Find the time optimal synthesis of steering points to the origin.

3. Consider the harmonic oscillator without friction
$$\begin{aligned} \frac{dx}{dt} &= \ y \\ \frac{dy}{dt} &= \ -x + u \end{aligned}$$
with control constraint by $|u| \leq 1$. Consider the origin 0 as the initial time. Prove that all the points on the boundary of the reachable set for $t > 0$ are exposed points.