Control and Optimal Control

Assignment 7, due January 25, 2012

1. Let $\mu_1, \mu_2, \ldots, \mu_n$ be n atom-less probability measures on a measure space M. Show that there is a partition $A_1, \ldots, A_n$ of M such that $\mu_i(A_j) = \frac{1}{n}$ for every i and j (interpretation: a cake with no raisins can be divided so that each piece is worth the same for every participant).
2. Let $K_1, K_2, \ldots$ be a sequence of compact sets all contained in the, say, unit ball of R^n. Show that that distance between $\frac{1}{j}(K_1 + \cdots + K_j)$ and $\frac{1}{j}(coK_1 + \cdots + coK_j)$ tends to zero as $j \to \infty$ (here coK is the convex hull of K, and the distance is taken in the Hausdorff sense). Can you estimate this distance?
3. Show that the reachable set of a control system $\frac{dx}{dt} = Ax + Bu$ with $u \in U(x)$ may not be convex even if the set-valued map $U(x)$ has compact sets and is continuous as a function of x.
4. What would happen if the set valued map $U(x)$ in the previous exercise has a convex graph?