Endogenously Dynamically Complete Equilibria for Financial Markets: The General Case

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Abstract

Only recently, existence of equilibrium in a continuous-time securities market in which the securities are endogenously dynamically complete has been proved. Given the fundamental nature of this result various extensions have been proposed. In the present paper we prove all results under optimal (much more general and natural) conditions. Namely, we only assume quasi-analyticity rather than analyticity of the basic economic ingredients, and we prove everything based solely on this hypothesis.

In order to put our work in the right perspective I am going over a very simple example. Let us assume that we have a stock and a money account. The stock follows

$$S \begin{array}{l} \nearrow \; uS \quad \text{with prob } q \\ \searrow \; dS \quad \text{with prob} 1-q \end{array}$$

where $0 < d < 1 + r < u$ (r interest rate). If we need to price the option

$$\mathcal{O} \begin{array}{l} \nearrow \; \mathcal{O}_u = \max[0,\ uS - K] \quad \text{with prob } q \\ \searrow \; \mathcal{O}_d = \max[0,\ dS - K] \quad \text{with prob} 1-q \end{array}$$

we trade in order to create a portfolio (Δ units of stock and B units of money)

$$\Delta\, uS + (1+r)\,B \qquad \text{prob } q$$

$$\Delta S + B$$

$$\Delta\, dS + (1+r)\,B \qquad \text{prob } 1-q$$

if the portfolio replicates the option then we must have

$$\begin{cases} \Delta\, uS + (1+r)\,B = \mathcal{O}_u \\ \Delta\, dS + (1+r)\,B = \mathcal{O}_d \end{cases}$$

solving (always possible if det $\begin{bmatrix} Su & 1+r \\ Sd & 1+r \end{bmatrix} \neq 0$) we obtain

$$\begin{cases} \Delta = \frac{\mathcal{O}_u - \mathcal{O}_d}{(u-d)S} \\ B = \frac{d\mathcal{O}_u - u\mathcal{O}_d}{(u-d)r} \end{cases}$$

so we get

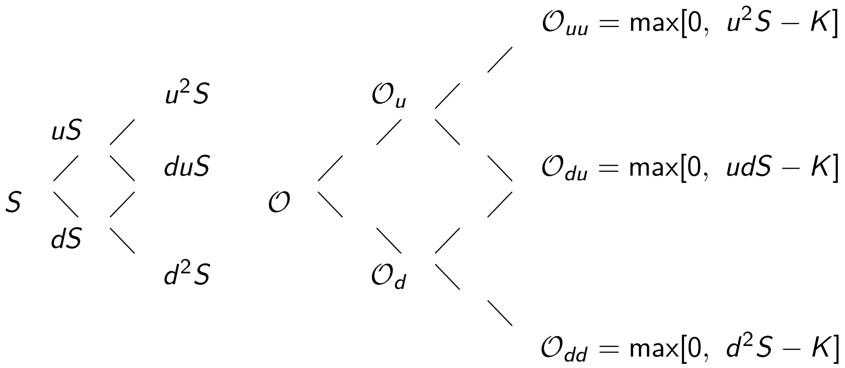
$$\mathcal{O} = \Delta S + B = [p\mathcal{O}_u + (1-p)\mathcal{O}_d] / (1+r)$$

where

$$p = \frac{1+r-d}{u-d}$$

$$1-p = \frac{u-1-r}{u-d}$$

2 -period Binomial Tree



To price we note that when there is one period left we have

$$\mathcal{O}_u = [p\mathcal{O}_{uu} + (1-p)\mathcal{O}_{ud}]/(1+r)$$

and

$$\mathcal{O}_d = [p\mathcal{O}_{du} + (1-p)\mathcal{O}_{dd}]/(1+r)$$

and also we observe that

$$\mathcal{O} = [p\mathcal{O}_u + (1-p)\mathcal{O}_d]/(1+r)$$

with

$$\mathcal{O}_{ud} = \mathcal{O}_{du}.$$

Main point: As long we assume the right conditions on the dynamic of the stock we can price everything. This means

$$\det \begin{bmatrix} Su & 1+r \\ Sd & 1+r \end{bmatrix} \neq 0, \det \begin{bmatrix} Su^2 & 1+r \\ Sud & 1+r \end{bmatrix} \neq 0, \det \begin{bmatrix} Sud & 1+r \\ Sd^2 & 1+r \end{bmatrix} \neq 0$$

(I know! the first implies the second and third but please wait...)

Pricing, Consumption and Investment It is useful also to think of pricing in relation to investment, the possibility of consumption and the presence of different agents in the market. In these cases we need to think of two main new features

- The possibility to give up present consumption for future consumption and the presence of many agents
- Resources are in general limited

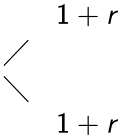
Typically we model these aspects by

- 1. A finite number of agents with von Neumann-Morgenstern utility functions
- 2. Presence of markets for goods and for assets
- 3. Focusing on equilibrium i.e. **agents maximize their utilities and all markets clear**

The key novelty is that the price for the asset is given by market clearing. Let's see an example.

Example Given $\{X_i\}_{i=1}^{N}$ i.i.d. with $\mathbf{P}(X_i = 1) = \mathbf{P}(X_i = -1)$ we consider the standard binary tree of length N

1. At each node the agents can trade an asset (short-lived) which pays the next period in this way



and they can trade a long lived asset A which pays only at the very end (period N) in this way

$$A(\omega, n) = \begin{cases} 0 & \text{if } n \neq N \\ e^{\frac{\sum^N X_i(\omega)}{\sqrt{N}}} & \text{if } n = N \end{cases}$$

2. There are two agents with payoffs given by

$$U_1(c) = \mathbf{E}\sum_{k=0}^{N} c_k(\omega)^{\frac{1}{2}} \text{ and } U_2(c) = \mathbf{E}\sum_{k=0}^{N} c_k(\omega)^{\frac{1}{3}}$$

At the initial node they have one unit of endowment each and at each successor node they have endowment as

agent 1	*agent* 2
1	0
0	1

Agent one owns the only available unit of the long lived asset. Note that at the very end the determinant condition is satisfied as we have

$$\begin{array}{cc} Se^{\frac{1}{\sqrt{N}}} & 1+r \\ Se^{\frac{-1}{\sqrt{N}}} & 1+r \end{array}$$

but, at each node, each agent faces

$$S_{up} \quad 1+r$$

$$S_{down} \quad 1+r$$

Main Question: Will prices of assets at equilibrium satisfy the determinant condition?

This question is crucial since if we want to price an option

$$\mathcal{O}_u = \max[0,\ S_{up} - K] \qquad \text{with prob } q$$

$$\mathcal{O}$$

$$\mathcal{O}_d = \max[0,\ S_{down} - K]] \quad \text{with prob} 1 - q$$

we cannot proceed as before since we face a fundamental problem

Problem Are S_{up} and S_{down} such that we can solve

$$\begin{cases} \Delta\ S_{up} + (1+r)\,B = \mathcal{O}_u \\ \\ \Delta\ S_{down} + (1+r)\,B = \mathcal{O}_d \end{cases} ?$$

Of course, we need that

$$\det \begin{bmatrix} S_{up} & 1+r \\ S_{down} & 1+r \end{bmatrix} \neq 0.$$

Please note the following:

- The determinant condition must be true at each node
- The agent's choice for consumption/investment depends only on the

$$span \left\langle \begin{bmatrix} S_{up} \\ S_{down} \end{bmatrix}, \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} \right\rangle$$

- If the determinant condition holds we say that the market is **COMPLETE** and (trivial! but important to keep in mind)

$$span \left\langle \begin{bmatrix} S_{up} \\ S_{down} \end{bmatrix}, \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} \right\rangle = span \left\langle \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\rangle$$

Continuous-Time: Advantages

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- The possibility to use more general processes
- The possibility to use stochastic calculus and PDE (ex. Feynman-Kac)
- The possibility to price very general classes of derivatives

Continuous-Time and Equilibrium: Peculiarities and Problems

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  Definition 2 *An Arrow-Debreu market is a market where all the Arrow-Debreu secuities are available at every node.*

Continuous-Time and Equilibrium: Peculiarities and Problems cont'd

- There is a sharp difference between an Arrow-Debreu market and a securities market. In the former, agents are allowed to trade a complete set of Arrow-Debreu contingent claims (i.e. standard basis of $\mathbf{R^n}$). This is impossible in a continuous-time market with a finite number of instruments.

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- With dynamic completeness (i.e. **determinant condition satisfied at each node**) in a securities market we achieve all the consumption allocations that could be achieved in an Arrow-Debreu market.
- A necessary condition for dynamic completeness is that the market is potentially dynamically complete. This means, if the uncertainty is driven by a multi-dimensional Brownian motion, that the number of linearly independent securities is at least one more than the dimension of this Brownian motion.

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Review of the Literature

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- Various extensions have been proposed for example Hugonnier, Malmud and Trubovitz; Kramkov and Prediou; Riedel and Herzberg.
- **Crucial role of analyticity.** In particular, explicit assumption of analyticity of the transition probabilities of the underlying Ito processes. It is desiderable that this last fact is proved from the basic economic data, rather than assumed.

What is the strategy for those papers?

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- ★ Construct Arrow-Debreu equilibrium for correspondent exchange economy
- ★ Show that prices are analytic
- ★ Assume a nondegeneracy condition on the final payoffs and exploit the fact that, for analytic function, being zero on an open set means being zero everywhere

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★ **Quasi-analyticity** is the right notion!

- A quasi-analytic class of functions is a generalization of the class of real analytic functions based upon the following fact. *If f is an analytic function on an open set Ω and at some point $x \in \Omega$ $f(x)$ and all of its derivatives $f^{(n)}(x)$ are zero, then f is identically zero on the open set.* **Quasi-analytic classes are much broader classes of functions for which this statement still holds true.**

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- Eventually, prices are going to satisfy the following (p_C comes from equilibrium)

$$\frac{\partial p_{Aj}}{\partial t} = -A(t,x,D)p_{A_j} - p_C(t,x)g_j(t,x) \tag{1}$$

with the boundary condition

$$p_{Aj}(T,x) = G_j(T,X(T))p_C(T,x) \tag{2}$$

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Fact: Kannai and Raimondo can use a new result about linear parabolic equations with quasi-analytic coefficients.

ref. *Quasi-analytic Solutions of Linear Parabolic Equations* forth. Journal d'Analyse Mathematique

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$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dB_t.$$

with $b(t, x)$ and $\sigma(t, x)$ quasi-analytic functions of $(t, x) \in (0, T) \times \mathbf{R}^K$ and continuous on the closure.

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- The agent's utility function is given by

$$U_i(c) = E_\mu \left[\int_0^T h_i(c_i(t,\cdot), X(t,\cdot))dt + H_i(c_i(T,\cdot), X(T,\cdot)) \right]$$

- There are $K+1$ securities $A_0, A_1, \ldots, A_K$; security $j = 0, ..., K$ is in net supply $\eta_j \in [0,1]$. Security j pays dividends (measured in consumption units) at a flow rate

$$D_j(t,\omega) = g_j(t, X_t)$$

at times $t \in [0, T)$, and a lump dividend $D_j(T,\omega) = G_j(X_T)$ at time T. We assume that $g : [0, T] \times \mathbf{R}^K \to \mathbf{R}_+^K$ is quasi-analytic on $(0, T) \times \mathbf{R}^K$ and that G_j is locally in L^2.

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- **Non Degeneracy Assumption**

There is an open set $V \subset \mathbf{R}^K$ such that $G_0(T,x) > 0$ for all $x \in V$ and for $j = 1, \ldots, J$ and $i = 1, \ldots, I$

$$G_j, F_i \in C^1(V) \text{ and } \forall x \in V \ \mathrm{rank} \begin{pmatrix} \left.\frac{\partial(G_1/G_0)}{\partial X}\right|_{(T,x)} \\ \vdots \\ \left.\frac{\partial(G_J/G_0)}{\partial X}\right|_{(T,x)} \end{pmatrix} = K \qquad (3)$$

- Agent i is initially endowed with deterministic security holdings $e_{iA} = (e_{iA_0}, \ldots, e_{iA_J}) \in \mathbf{R}^{J+1}$ satisfying

$$\sum_{i=1}^{I} e_{iA_j} = \eta_j$$

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- The definition of budget constraints, trading strategies and equilibrium are the standard ones for models where the securities are priced *cum dividend*. Of course, we also use the following standard

Definition

A Radner equilibrium is a set of price processes $(p_C, p_{A_0}, p_{A_1}, ..., p_{A_K})$, a consumption allocation $(c_i)_{i=1}^I$ and a set of strategies $((z_{iA_0}, ..., z_{iA_K}))_{i=1}^I$ such that

(a) The plan c_i maximizes U_i over the budget set and is financed by $(z_{iA_0}, ..., z_{iA_K})$,

(b) All markets clear.

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Theorem

The continuous-time finance model just described has an equilibrium, which is Pareto optimal. The equilibrium pricing process is effectively dynamically complete, and the admissible replicating strategies are unique. Moreover the prices for assets and goods are quasi-analytic.

Our proof is in four steps.

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Step 4 Show the completeness of prices via result on Parabolic Equations

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THANK YOU!