A Theory of Social Interactions and Formation of International Alliances

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Two major parts:in this research project:

- Societal Diversity and Heterogeneity

- Social Interactions

and, naturally,

social interactions in diverse environments

Diversity.

Types of diversity; ethnic, historical, geographical, economic, historical, genetic, ideological...

Is diversity good or bad?

As YES and NO. (But even if it is good, there is no free lunch for diversity.)

“Before we can change direction, we have to question many of the assumptions underlying our current philosophy.

Assumptions like bigger is better; you can’t stop progress; no speed is too fast; globalization is good.

Then we have replaced it with some different assumptions:

small is beautiful; roots and traditions are worth preserving; variety is the spice of life; the only work worth doing is a meaningful work; biodiversity is the necessary pre-condition for human survival.” Robert Bateman.

Saxenian (1996, 1999) argued that the success of Silicon Valley in 80s and 90s was due to a diverse cultural and professional background of scientists and entrepreneurs.

Florida (2002), Florida and Gates (2001) examined the importance of diversity to high-tech growth. They ranked 50 US cities in terms of diversity (number of artists, foreign-born, homosexuals) and showed the success of more diverse ones (from San Francisco to Buffalo).

On the other hand, diverse societies, groups, countries, unions, families, may find it difficult to function due to different views, characterstics, approaches. European Union, Tragedy of Africa.

Measurement.

Suppose the society's total population is N.

There are K different attributes (languages, religion, ethnicity...)

Thus, there are K different groups, identified by the attributes.
In our setting, for every two groups i, j there is a distance $d(i, j)$.

The introduction of distances helps to address the group identification problem.

Example: Linguistic distances

Victor Ginsburgh and Shlomo Weber "How Many Languages Do We Need: Economics of Linguistic Diversity", Princeton University Press, 2011.

The distance matrix is based on cognate date collected by Isidore Dyen in the 1960s:

200 basic meanings (chosen by Swadesh (1952))
95 Indo-European speech varieties (languages and dialects)
For each meaning - there is a *cognate class* of different speech varieties that have an unbroken history of descent from common ancestral word.

For every two varieties, l and m, let n_{lm} and n^0_{lm} be the number of "cognate" and "non-cognate" varieties, respectively. Then the entry of the Dyen matrix is

$$d(l,m) = \frac{n^0_{lm}}{n^0_{lm} + n_{lm}}.$$

	IT	FR	SP	PT	GE	DU	SW	DA	EN
IT	0	0,20	0,21	0,23	0,73	0,74	0,74	0,74	0,7
FR	0,20	0	0,27	0,29	0,76	0,76	0,76	0,76	0,7
SP	0,21	0,27	0	0,13	0,75	0,74	0,75	0,75	0,7
PT	0,23	0,29	0,13	0	0,75	0,75	0,74	0,75	0,7
GE	0,73	0,76	0,75	0,75	0	0,16	0,30	0,29	0,4
DU	0,74	0,76	0,74	0,75	0,16	0	0,31	0,34	0,3
SW	0,74	0,76	0,75	0,74	0,30	0,31	0	0,13	0,4
DA	0,74	0,76	0,75	0,75	0,29	0,34	0,13	0	0,4
EN	0,75	0,76	0,76	0,76	0,42	0,39	0,41	0,41	0
LI	0,76	0,78	0,77	0,78	0,78	0,79	0,78	0,78	0,7
LA	0,78	0,79	0,79	0,80	0,80	0,80	0,79	0,80	0,8
SV	0,76	0,78	0,77	0,78	0,73	0,75	0,75	0,73	0,7
CZ	0,75	0,77	0,76	0,76	0,74	0,76	0,75	0,75	0,7
SL	0,75	0,76	0,75	0,76	0,74	0,75	0,74	0,73	0,7
PL	0,76	0,78	0,77	0,78	0,75	0,77	0,76	0,75	0,7
GR	0,82	0,84	0,83	0,83	0,81	0,81	0,81	0,82	0,8
RU	0,76	0,77	0,77	0,77	0,76	0,78	0,75	0,74	0,7
UKR	0,77	0,78	0,78	0,78	0,76	0,79	0,76	0,76	0,7

Genetic distances.

Genetic distances between Yugoslav republics

	Croatia	Bosnia and H	Serbia	Slovenia	Macedonia	Montenegro
Croatia	0	25.12	35.26	26.26	67.39	23.13
Bosnia and H	25.12	0	22.08	9.15	18.50	0
Serbia	35.26	22.08	0	15.36	82.92	8.50
Slovenia	26.26	9.15	15.36	0	33.12	14.44
Macedonia	67.39	18.50	82.92	14.44	0	0.21
Montenegro	23.13	0	8.50	14.44	0.21	0

Average distance 33 is on the level of Italy and France, but larger than Belgium and the Netherlands, Germany and Austria.

Social Interactions

Suppose, we need to decide which laptop to purchase and assume, for simplicity, that all options are reduced to two choices, PC and MAC. We rely on

intrinsic preferences and a potential benefit based on laptops' features, design, price and our prior computer experience.

However, we often invoke elements of "social interaction" that can manifest itself in two ways.

One is *local*, when we consult with our peers (colleagues, co-authors, friends, family members, neighbors).

Another is *global*, which represents the influence exerted by a global "market appeal", which could be positive or negative.

To provide a framework for the discussion, consider the following simple model with a finite set of players denoted by $N = \{1, 2, ..., n\}$.

X_i is a set of possible actions of player i (finite or infinite)
(a compact set in an Euclidean space)

The choices of individual strategies $x_i \in X_i$ for all $i \in N$ yield the $n-$dimensional strategies profile $\mathbf{x} = (x_1, x_2, ..., x_n)$

Profile $\mathbf{x}$ generates a partition $\pi(\mathbf{x})$ of N, where players i and j belong to the same group in $\pi(\mathbf{x})$ if their choices are identical, i.e., $x_i = x_j$.

We denote by $S^i(\mathbf{x})$ the group in $\pi(\mathbf{x})$ which contains player i.

The society is divided into peer groups, where each player i has a peer group $P^i \subset N$ that may influence i's choices.

The payoff $U_i(\mathbf{x})$ of player i, is the sum of three terms:

$$U_i(\mathbf{x}) = V_i(x_i) + \sum_{j \in P^i} W(d(i,j)) + H\left(x_i, |S^i(\mathbf{x})|\right).$$

The first term describes the intrinsic taste of player i for her chosen action x_i.

The other two terms, reflect various aspects of social interaction.

The second term represents bilateral social interactions of player i with her peer group.

The last term, which captures a *conformity* facet of social interaction and depends on the number of players who have chosen the same action x_i.

Nash equilibrium

Consider a model with a finite set of players denoted by $N = \{1, 2, ..., n\}$.

X_i is a set of possible actions of player i (finite or infinite)
Let $X = \Pi_{i=1}^{n} X_i$

Choices of individual strategies $x_i \in X_i$ for all $i \in N$ yield the $n-$dimensional strategies profile $\mathbf{x} = (x_1, x_2, ..., x_n)$

There are payoff (utility) functions $U_i : X \to \Re$

A profile $\mathbf{x}* = \left(x_1^*, x_2^*, ..., x_n^*\right)$ is a Nash equilibrium if the following inequality holds for every $i \in N$ and every $x_i \in X_i$:

$$U_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i-1}^* ..., x_n^*) \leq$$

$$U_i(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i-1}^* ..., x_n^*).$$

Result 1: If the set of strategies is finite, there is always a pure strategies Nash equilibrium.

Result 2: Let the set of strategies X be a compact set in $\Re^n$. Assume that the functions V_i (for every i) ,W and H (in the first argument) are upper semi-continuous. If, moreover, H is increasing (conformity!) in the seond argument, then there exists a pure strategies Nash equilibrium.

The conformity is needed if the strategy sets are infinite.

Example. There are two players, $i = 1, 2$, the strategy set is the interval $[0, 1]$ for both. For every pair of strategies x_1, x_2 the payoff function of player $i = 1, 2$ is given by

$$u_i(x_1, x_2) = \begin{cases} x_i & if \;\; x_i \neq x_j \\ x_i - 2 & if \;\; x_i = x_j \end{cases}$$

No Nash equilibria in pure strategies (Both would like to move to the right as far as possible but stay alone. This does not work).

The proof by using the method of potential functions (Rosenthal (1973), Monderer-Shapley (1996), Konishi-Le Breton-Weber (1999)).

We construct a "potential function" P on the set of all pure strategy profiles $P : \mathbf{x} \to P(\mathbf{x})$ and show that:

- P has a maximum (we need the compactness and continuity assumptions in the infinite case, in the finite case the maximum is guaranteed);

- every (local) maximum of P constitutes a Nash equilibrium.

That is, if $\mathbf{x}$ is not a Nash equilibrium, and there is a player i who would be better off by choosing the strategy y_i, then $P(\mathbf{x}) < P(\mathbf{x}_{-i}, y_i)$.

More specifically, we define the function P on $\prod_{i=1}^{n} X_i$:

$$P(\mathbf{x}) = \sum_{i=1}^{n} V_i(x_i) + \frac{1}{2} \sum_{j \in P^i} W(d(i,j)) +$$

$$\sum_{k=1}^{K} \sum_{r=1}^{|S_k|} H(x^k, r).$$

• Landscape Theory (Axelrod and Bennett (1993)). They consider European countries and "historical distances' and two blocs. There are two equilibria:

one is the exact partition into the Axis and Allies of World War II,
and another that separates the USSR, Yugoslavia and Greece from the rest of Europe.

• Iraqi wars (Kosenkova (2012)). Linguistic, ethic, and volume of trade distances.

• Coalitions of natural gas producers in the Middle East (Poyker (2012)). Various types of distances: One of the conclusions is the emergence of the following partition:

A bloc that consists of five countries:
Iran, Iraq, Oman, Kuwait, and UAE,
and three singletons: Saudi Arabia, Qatar and Yemen.