

THM: $P \subseteq P^F$ iff $P = NP$

$P \subseteq P^F \Rightarrow$ For each $S \in NP$,
consider the verification V
and let $R \equiv \{ \langle x, y \rangle : V(x, y) = \beta \}$.
Then $R \in PC \Rightarrow R \in PF$.

Decide S by finding a witness
(note: $S = S_R = \{ x : R(x) \neq \emptyset \}$)

$P = NP \Rightarrow$ For each $R \in PC$,
let $S \equiv \{ \langle x, y' \rangle : \exists y'' \langle x, y'' \rangle \in R \}$.

Then $S \in NP \Rightarrow S \in P$.

Find solution of x by
extending a prefix-solution bit by bit

Typical routine starts with y'
that is a prefix-solution

• If $\langle x, y'0 \rangle, \langle x, y'1 \rangle \notin S$,

then $y' \in R(x) \Rightarrow$ output y'

• If $\langle x, y'0 \rangle \in S$ for some $y'0$
then $y' \leftarrow y'0$

?
 $(x, y) \in S \Rightarrow \exists y'' \langle x, y'' \rangle \in R$
 $\perp \exists \emptyset (R(x) = \emptyset \text{ is not possible})$