Representation count, rational singularities of deformation varieties, and pushforward of smooth measures

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Topics

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$$\mathrm{Def}_{G,\Sigma_n} = \{(g_1, h_1, \dots g_n, h_n) \in G^{2n} | [g_1, h_1] \cdots [g_n, h_n] = 1\} = \\ = Hom(\pi_1(\Sigma_n), G),$$

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and, more generally, the map $\Phi := \Phi_G : G^{2n} \to G$ given by

$$(g_1, h_1, \ldots g_n, h_n) \mapsto [g_1, h_1] \cdots [g_n, h_n].$$

Representation count

Theorem (A.-Avni)

$$\exists \text{ constant } d \text{ s.t. } \#\{\pi \in \mathrm{irr} G(O) | \dim \pi \leq N\} \leq c(G) N^d$$

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- We in fact prove that the Dirichlet series

$$\zeta_G(2n) = \sum_{\pi \in \mathrm{irr} G} \dim \pi^{-2n}$$

converges for all $n \geq d'$.

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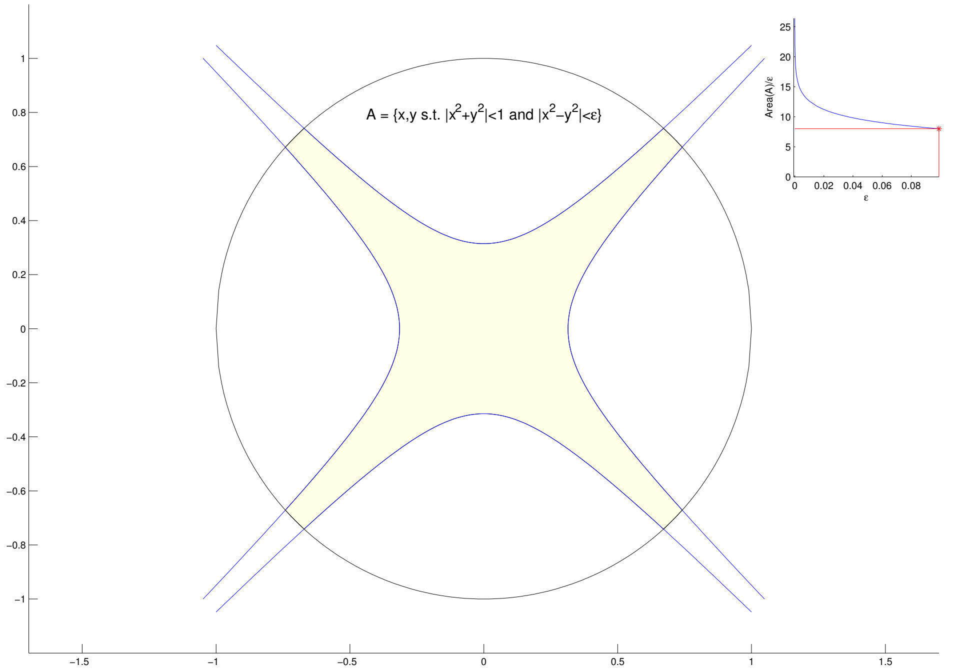
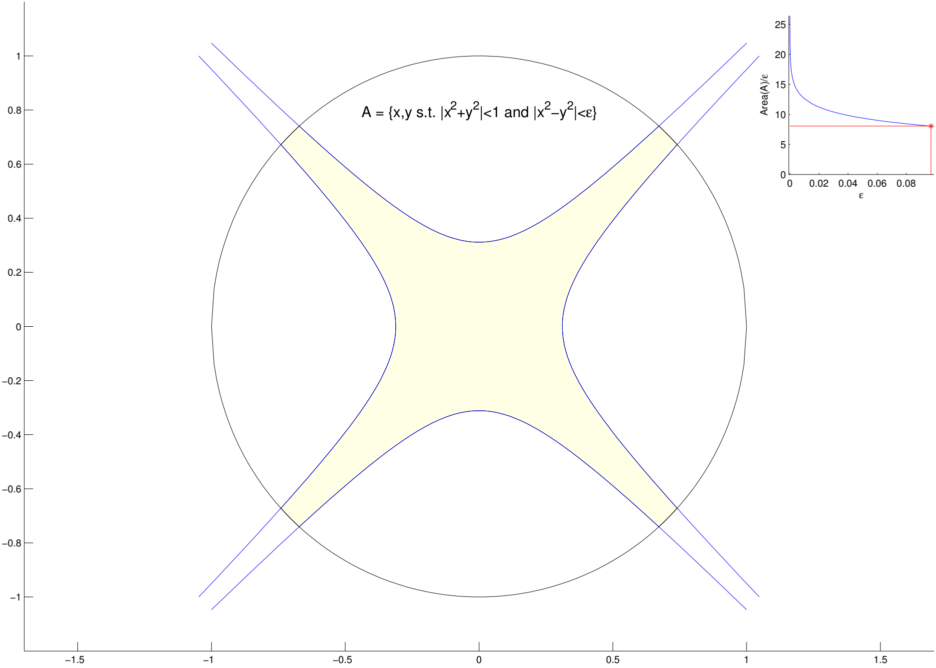
- We also have a converse result.
- ϕ is submersive (a.k.a. smooth) $\Rightarrow \phi_*(m)$ is smooth
- ϕ is (locally) dominant $\Rightarrow \phi_*(m)$ has L^1 density (Radon–Nikodym)



A = {x,y s.t. |x2+y2|<1 and |x2−y2|<ε}
Area(A)/ε
ε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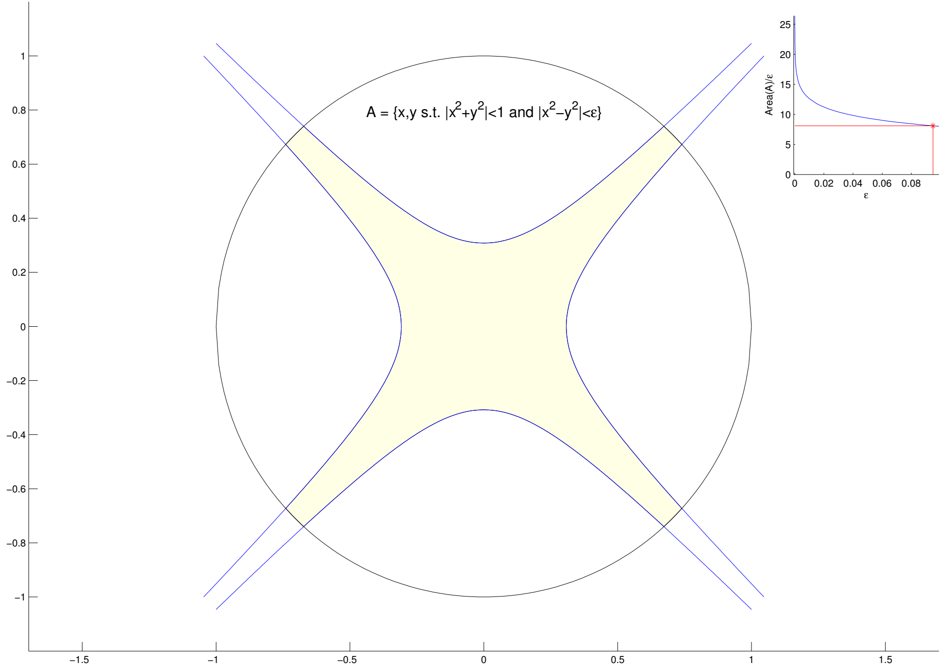
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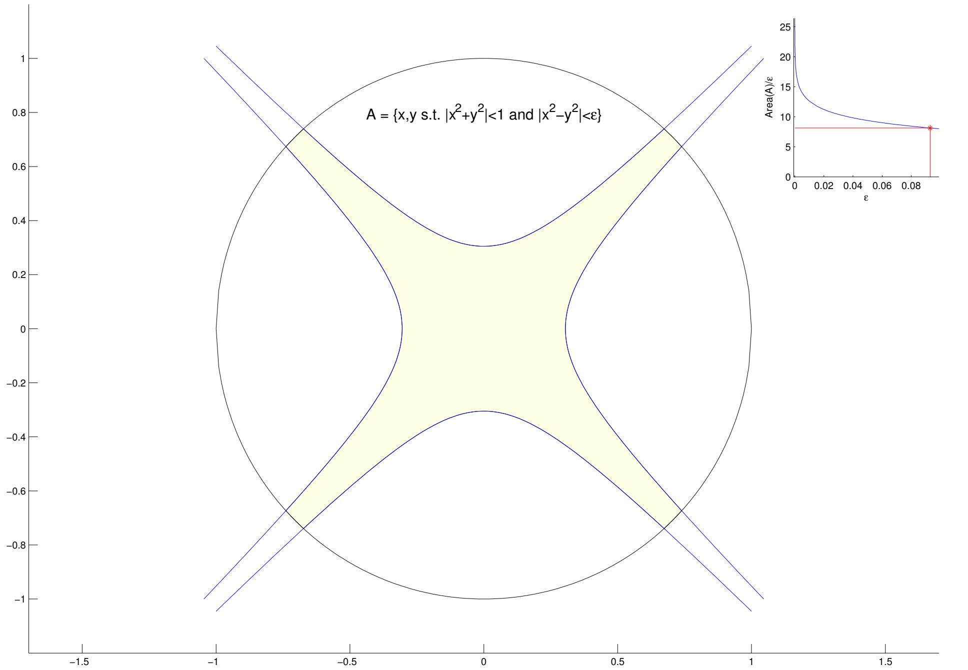
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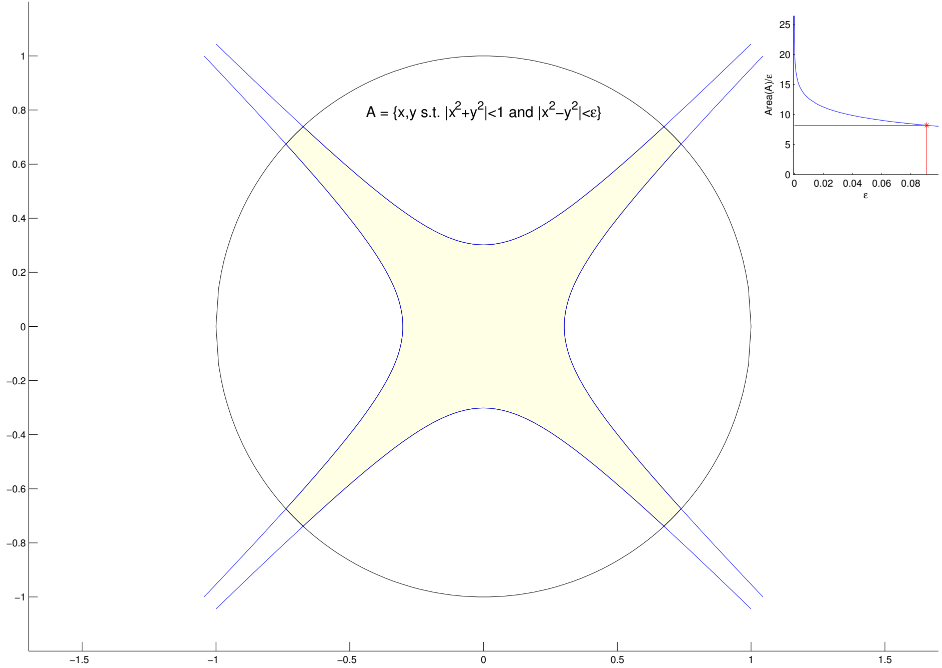

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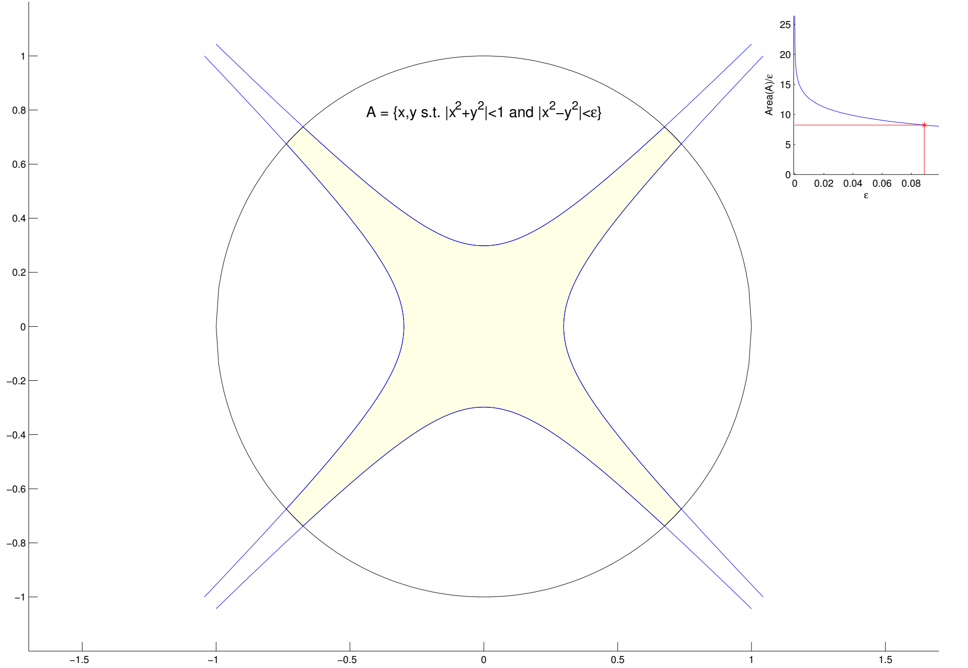
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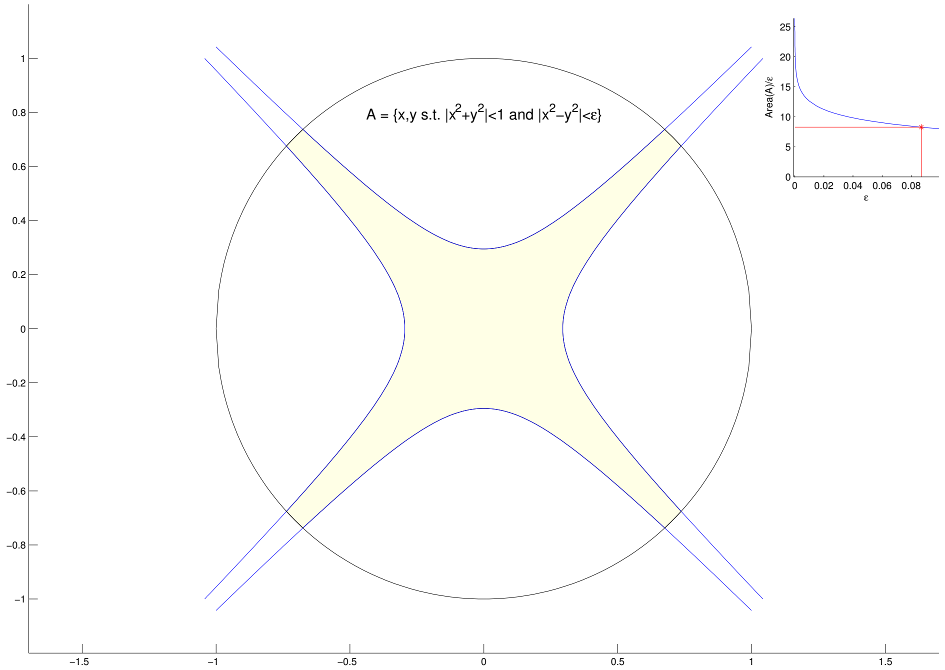

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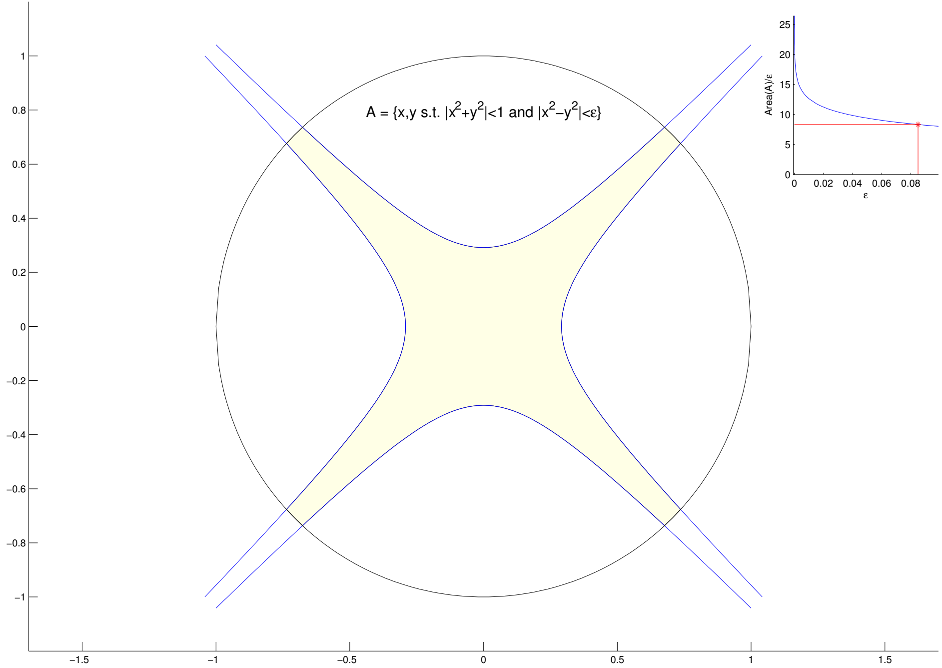
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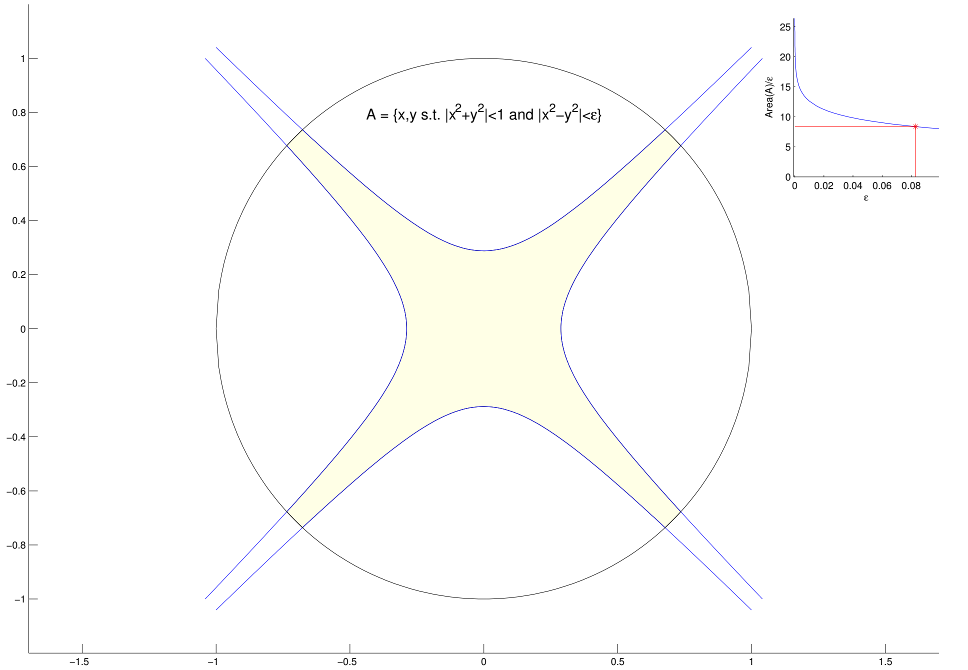

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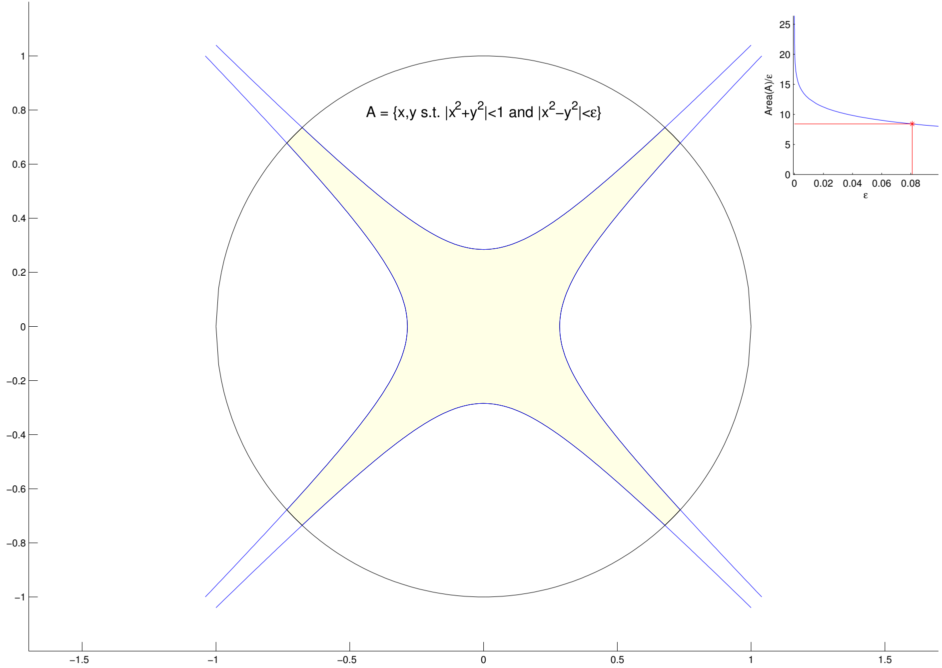
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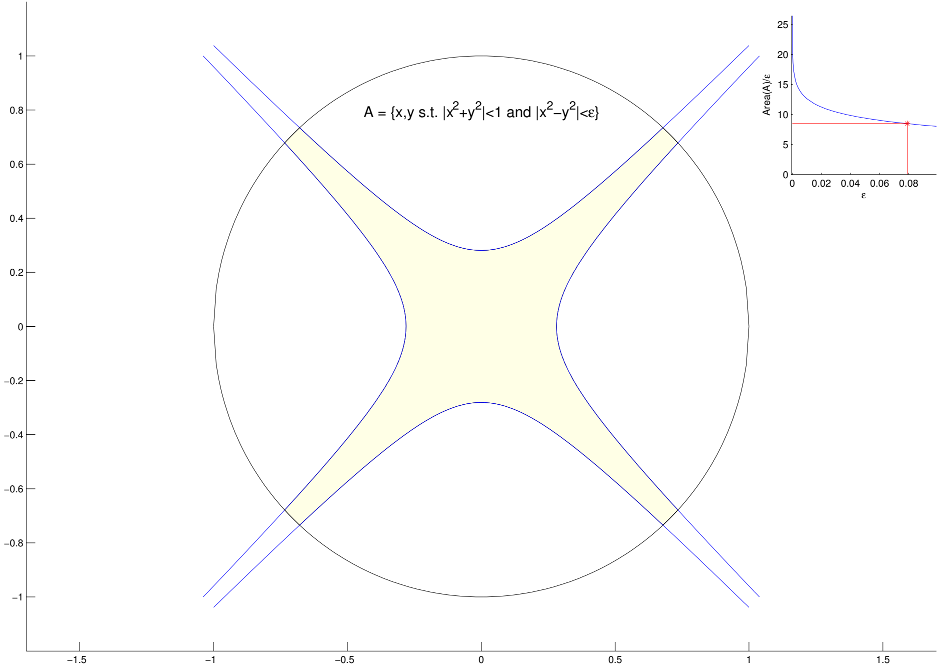

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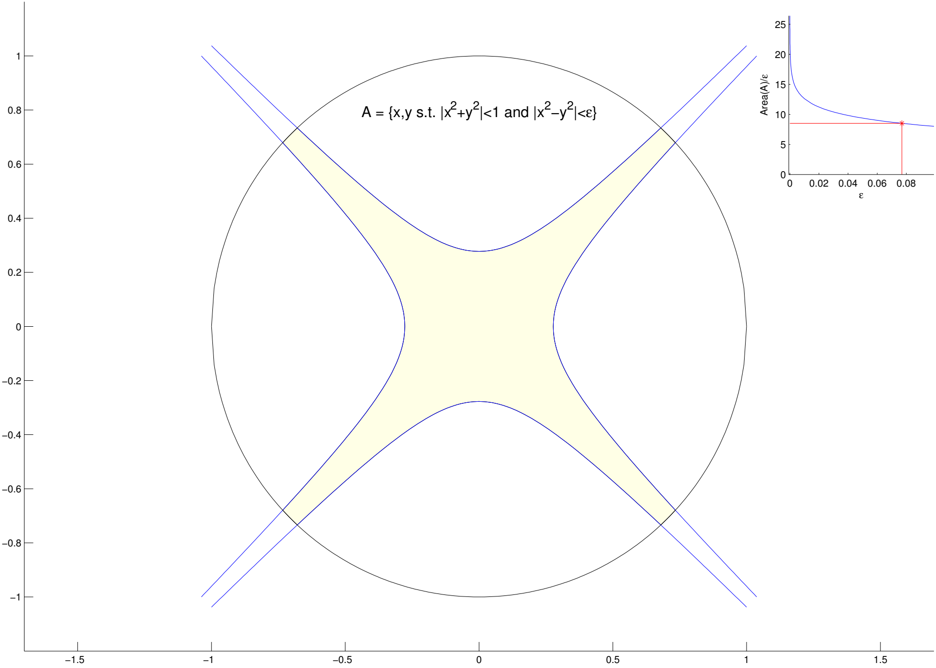
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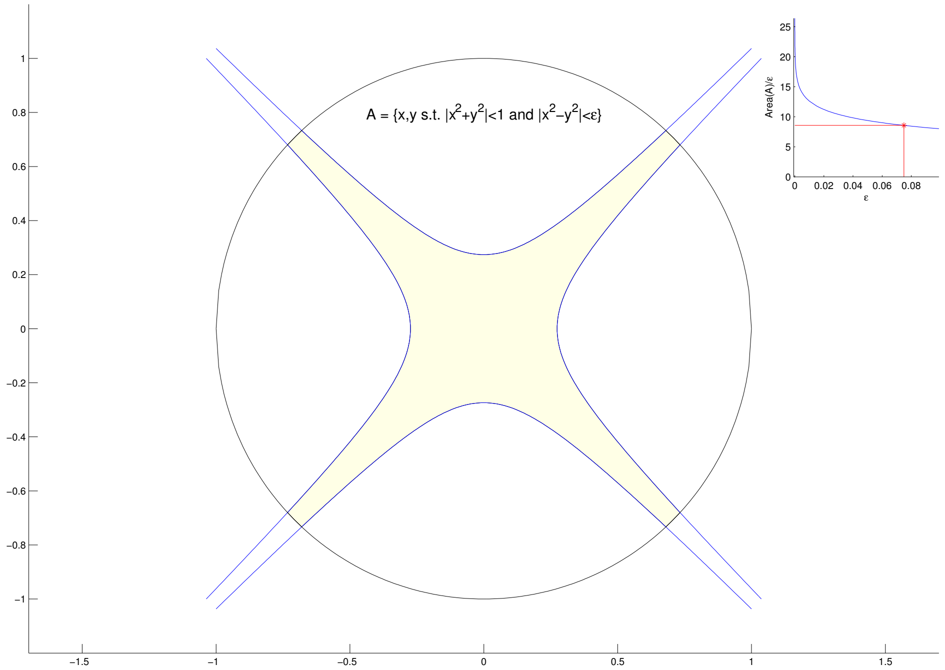
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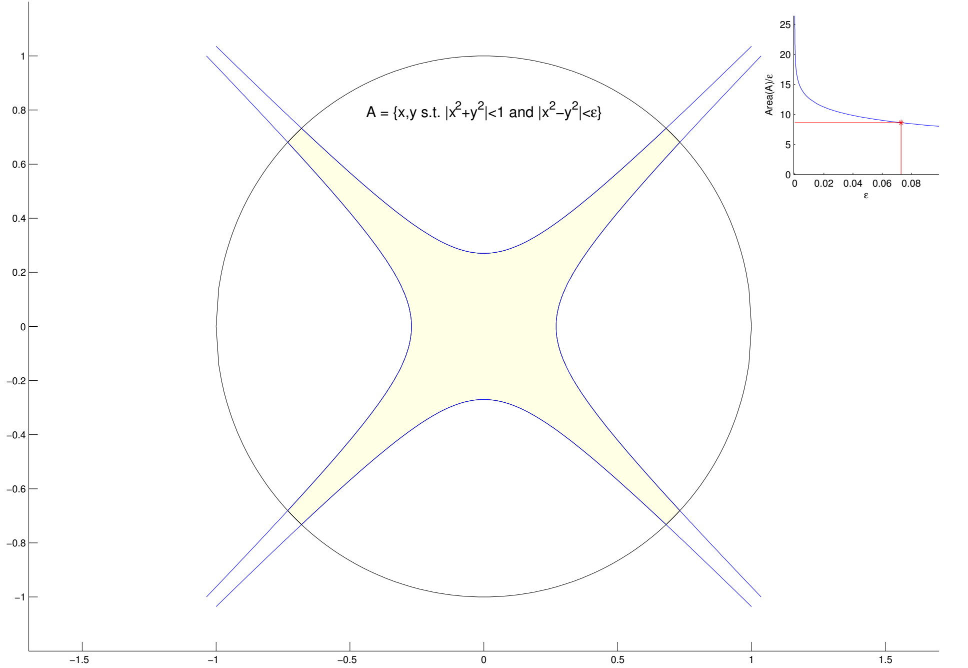

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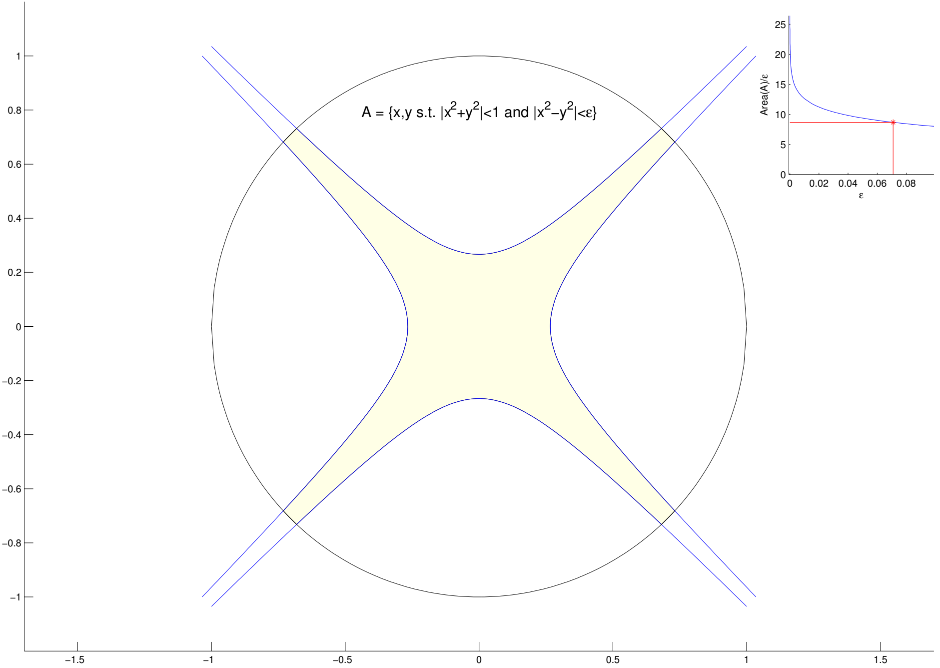

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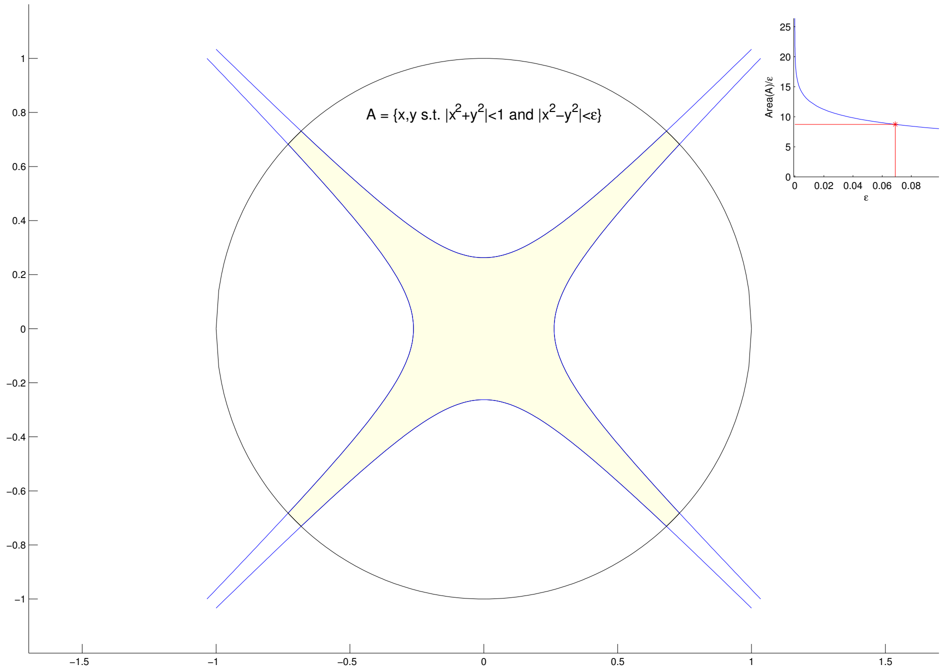

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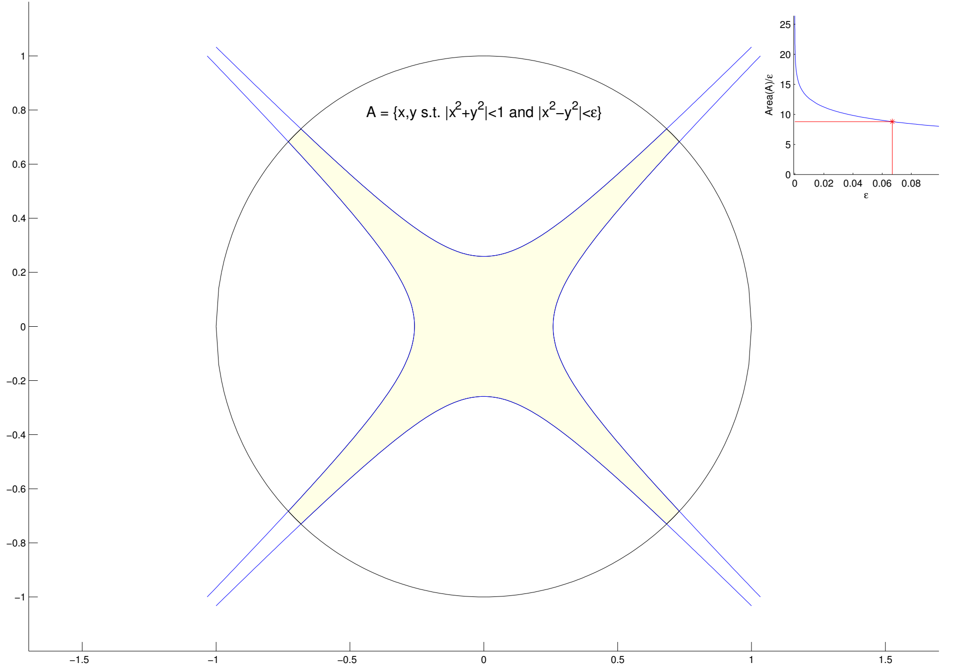
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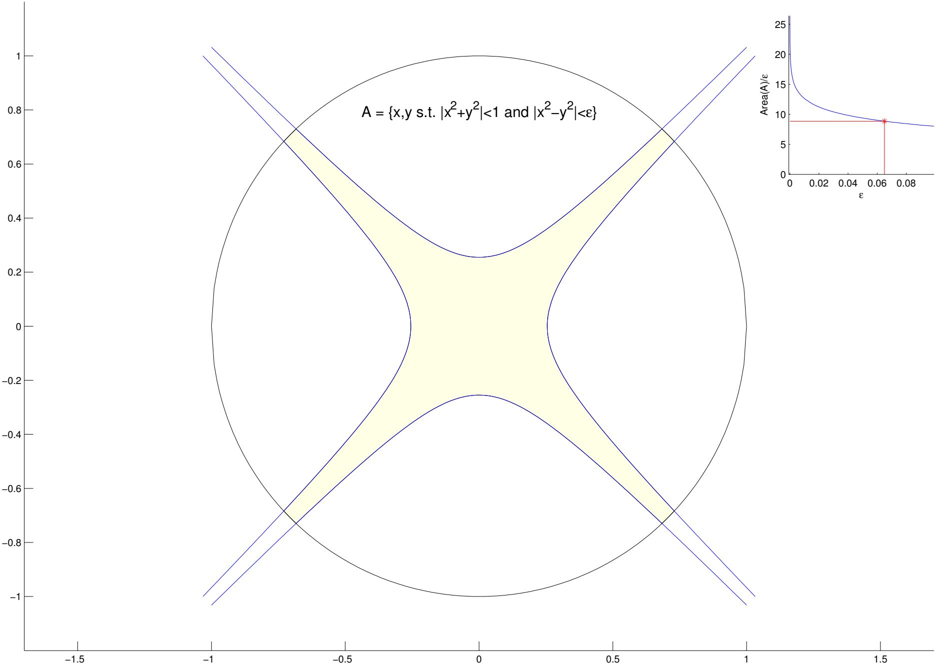

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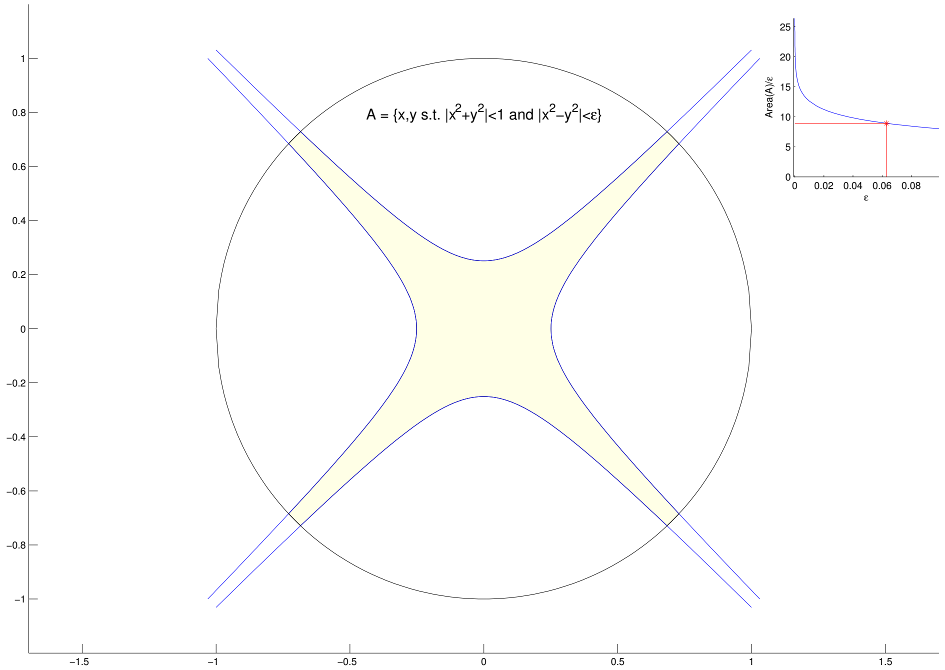

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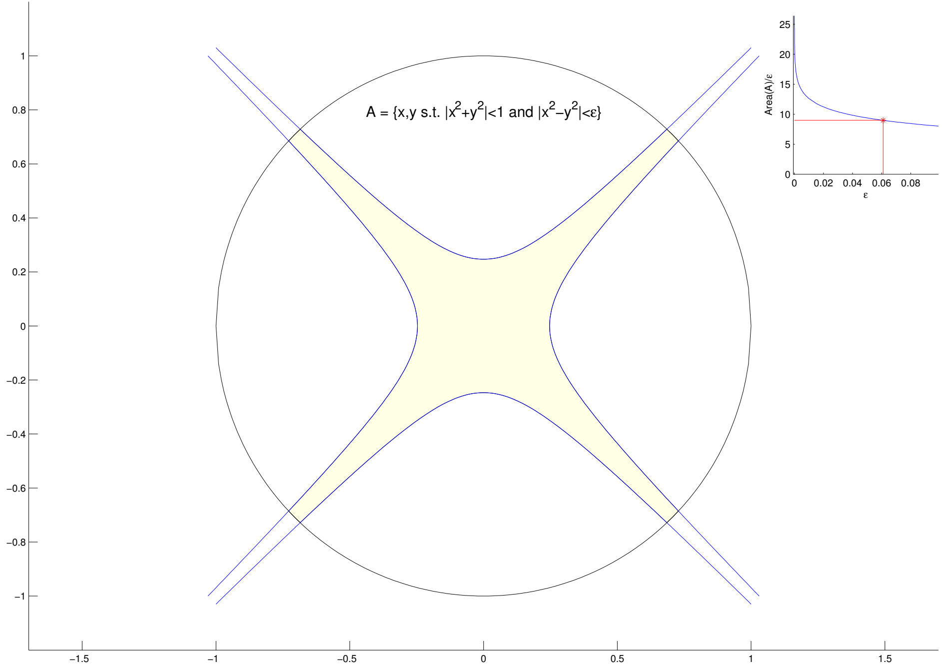
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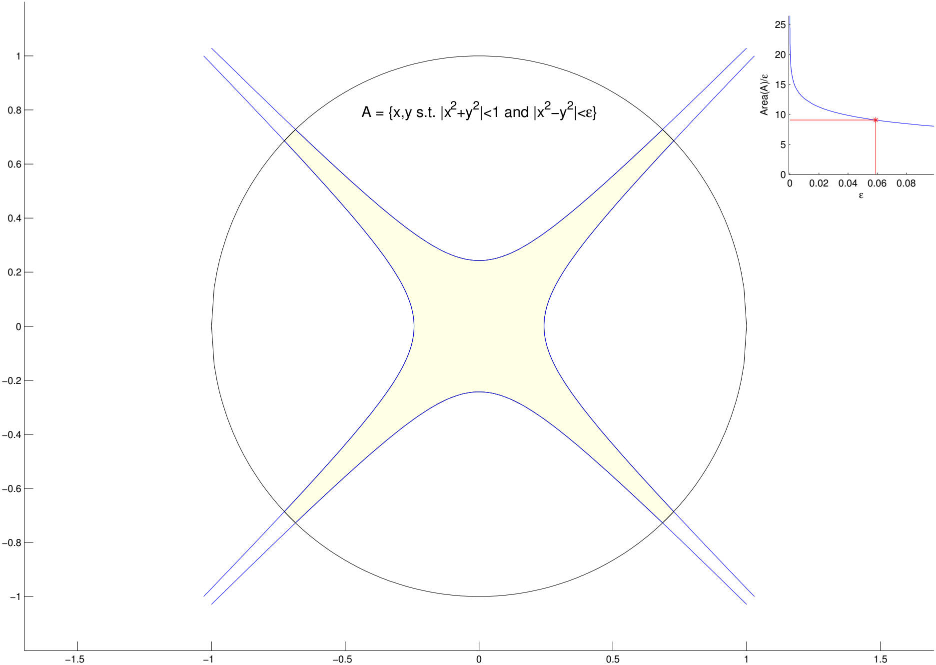

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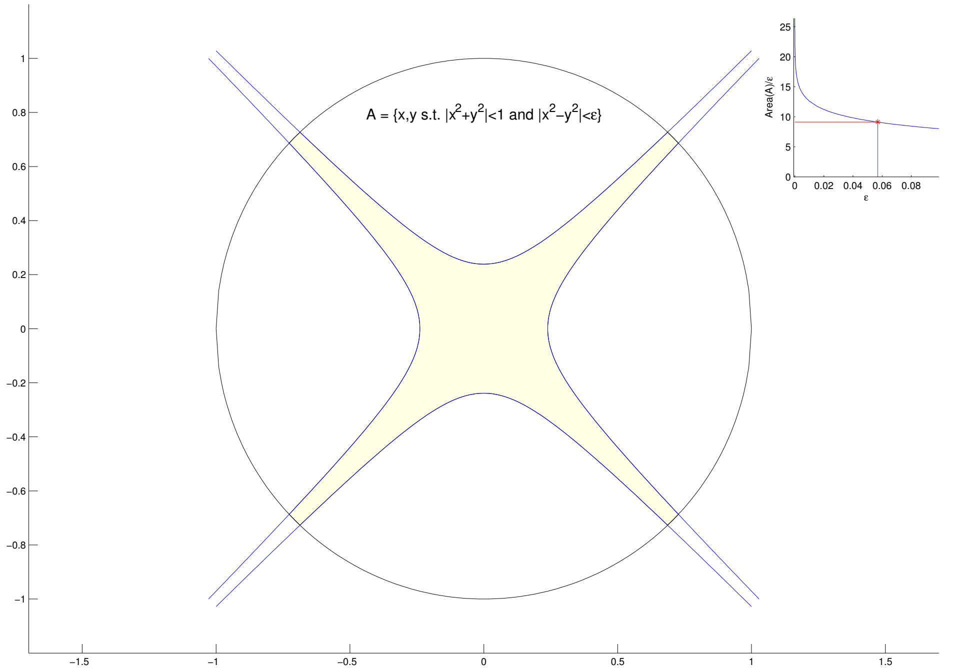

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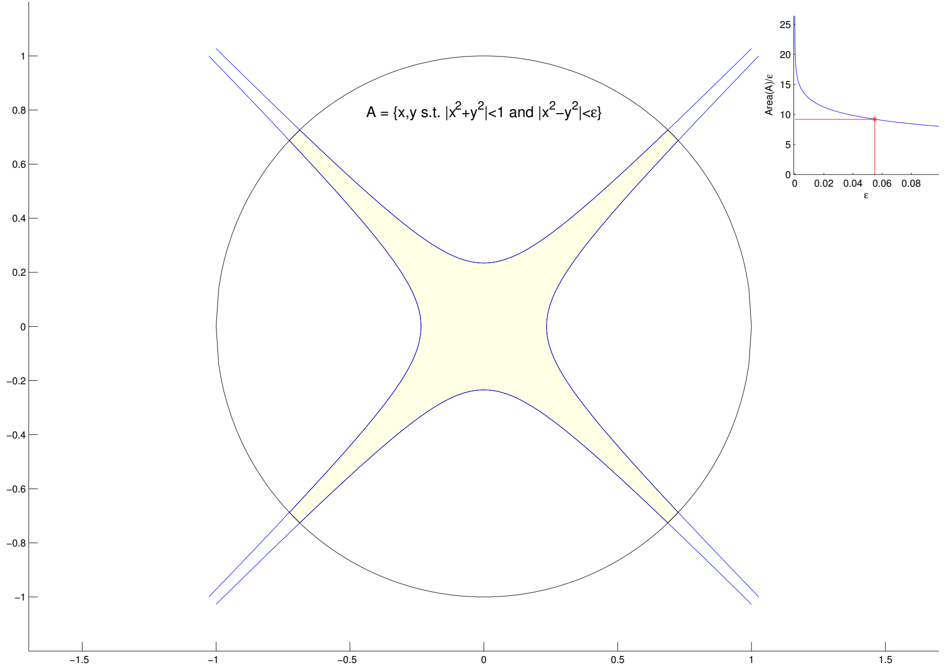
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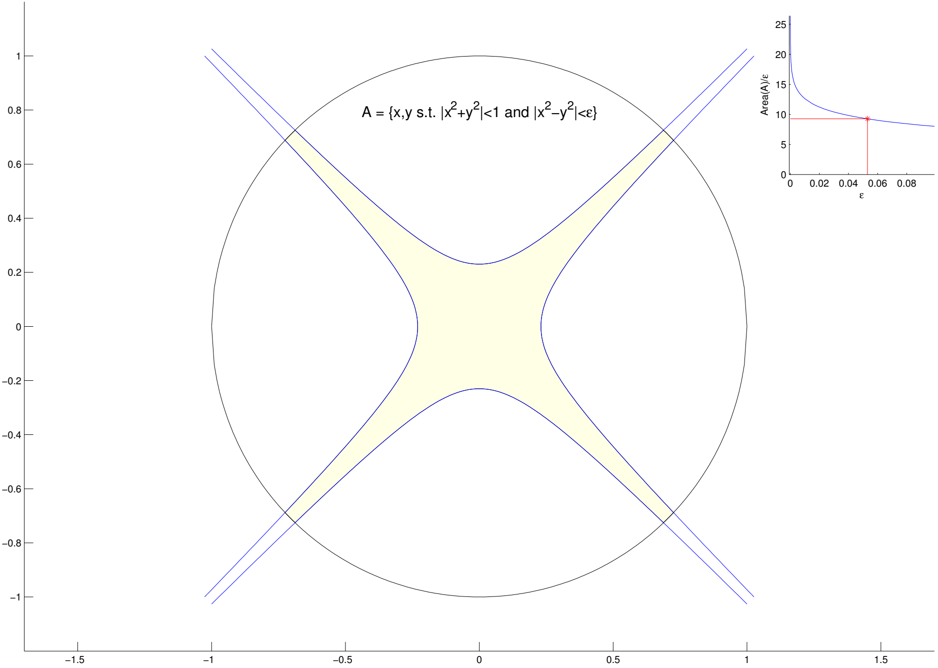

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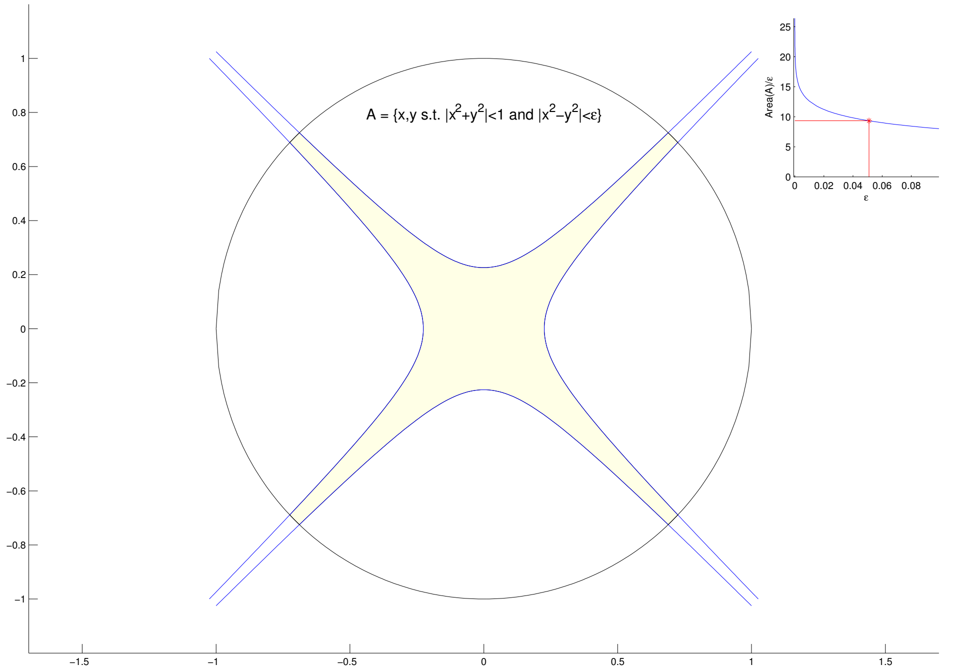

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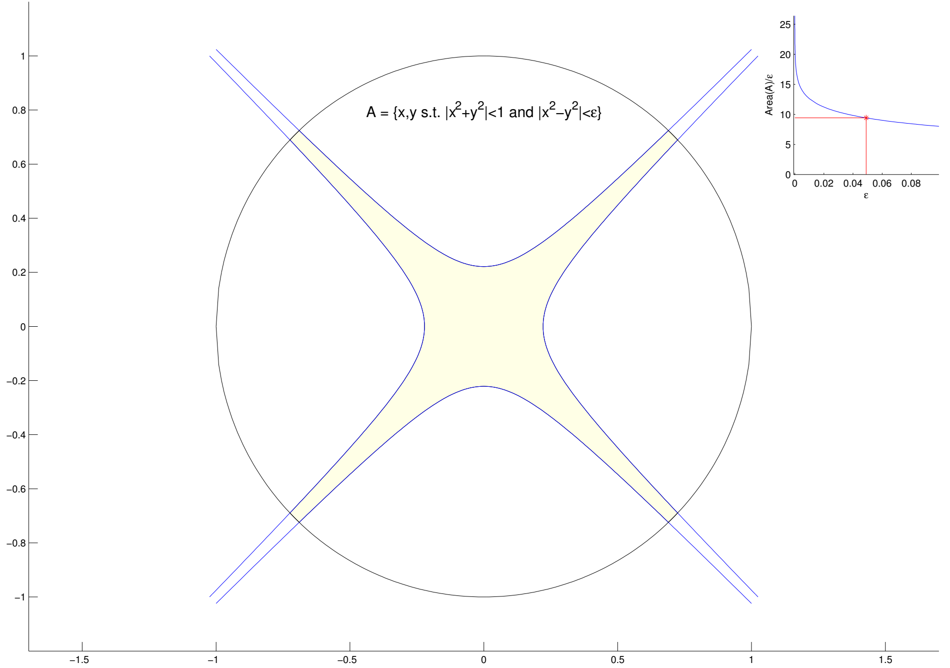
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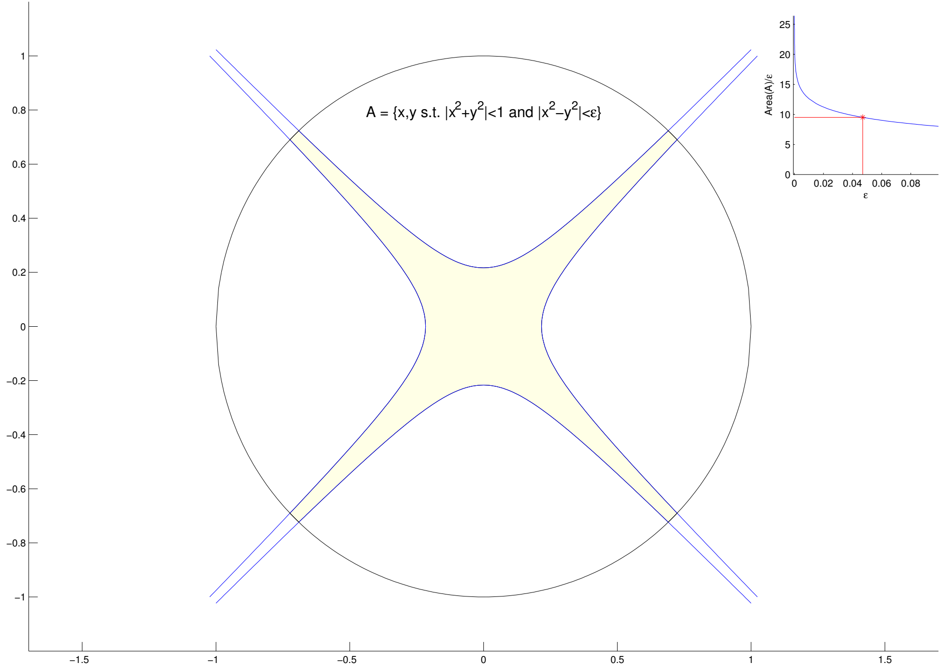

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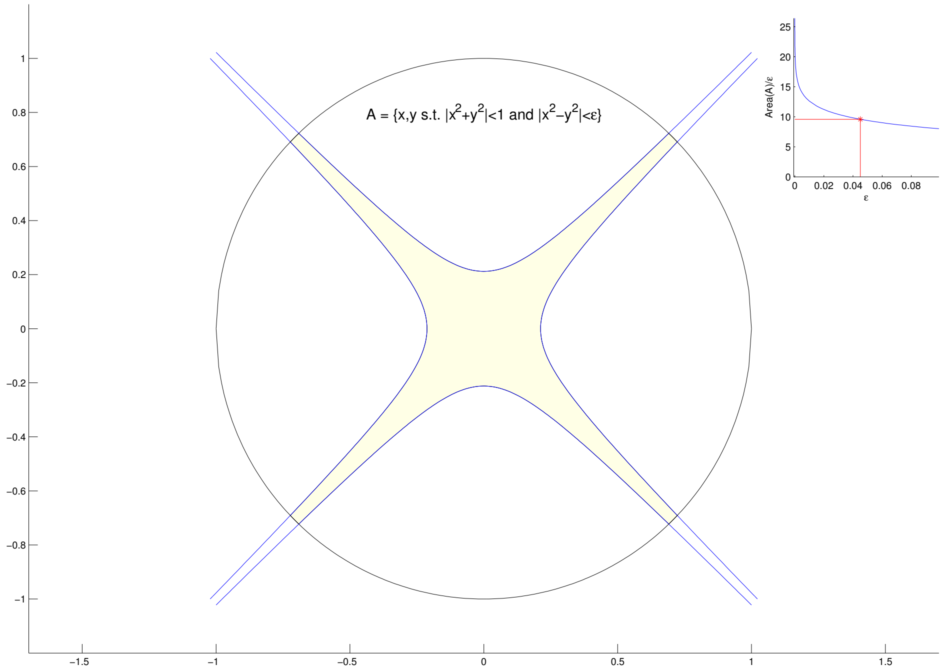

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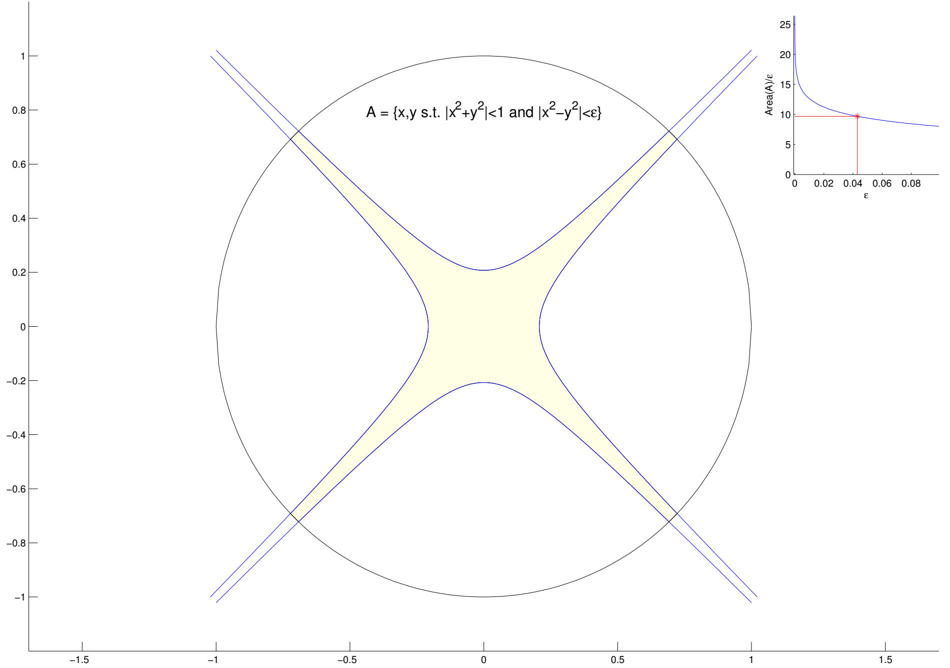
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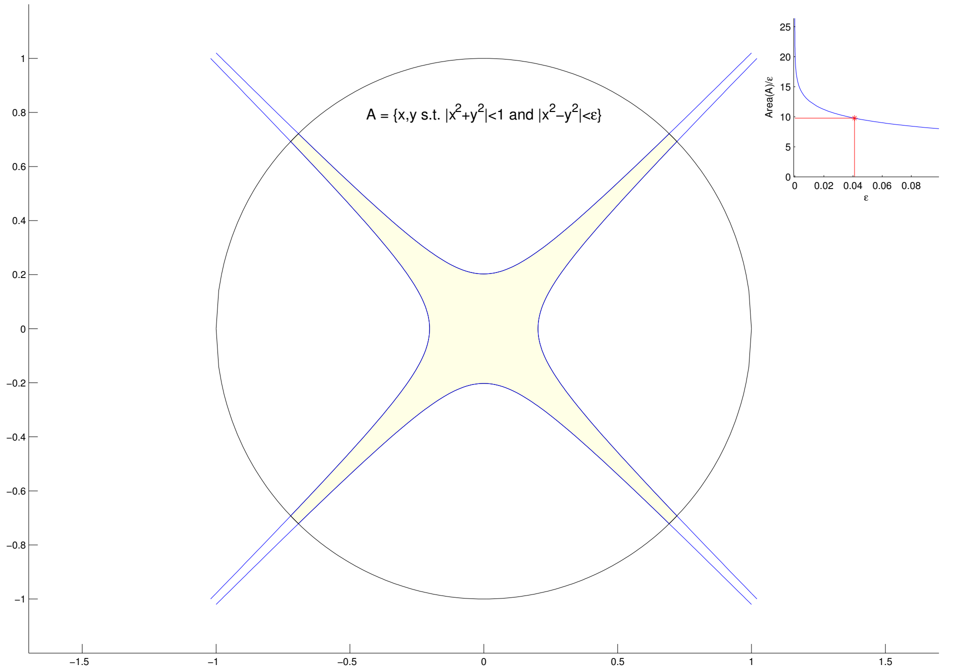

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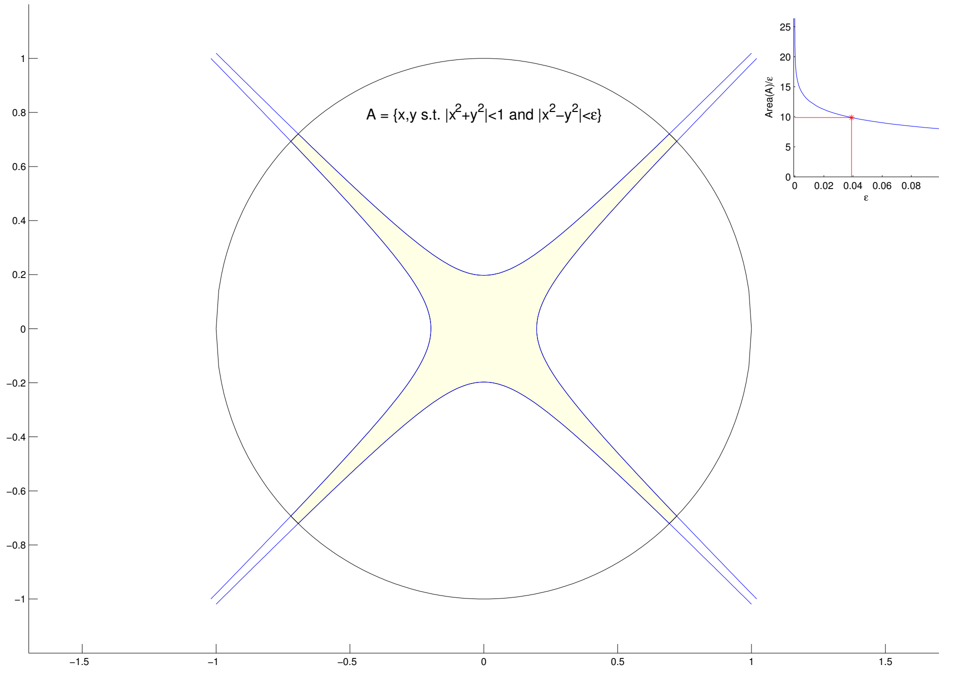

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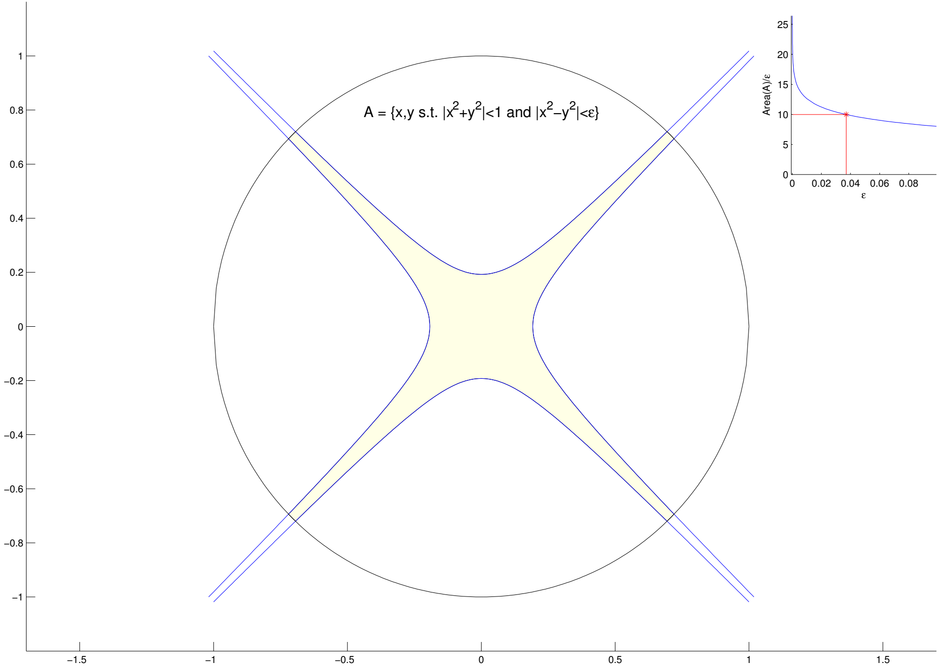
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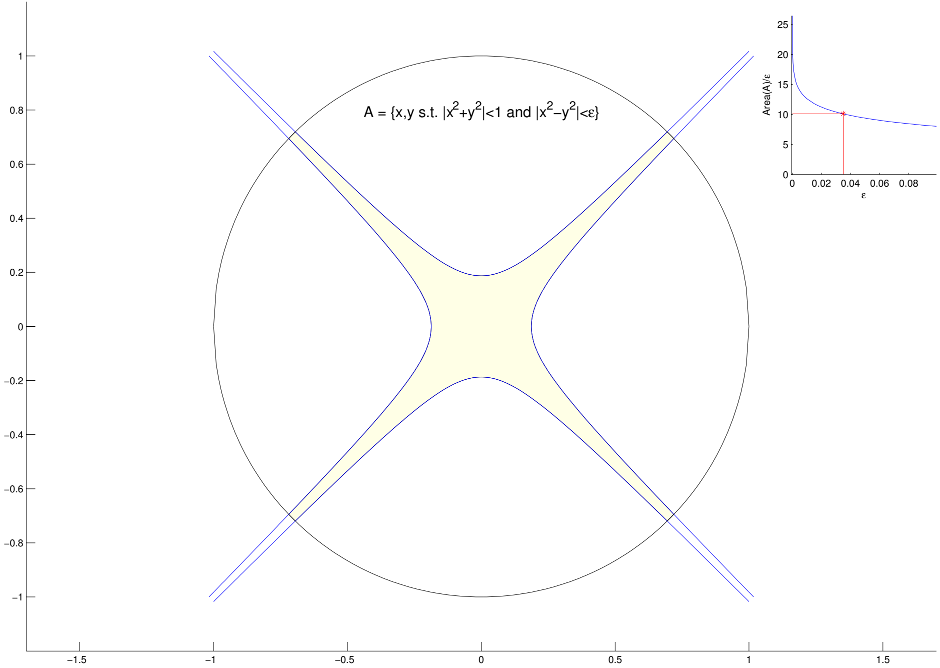

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0.6
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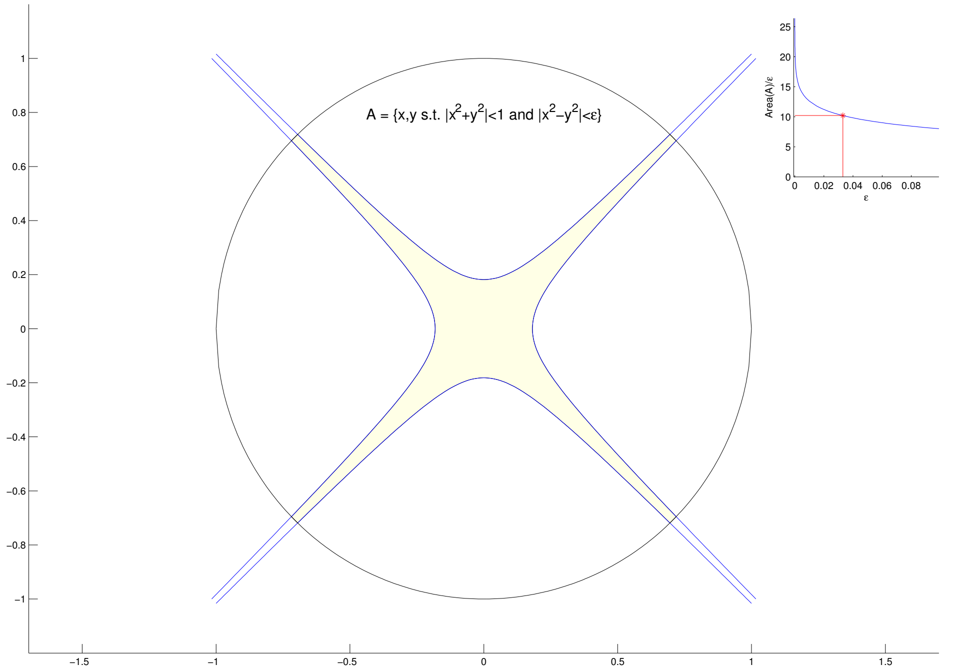

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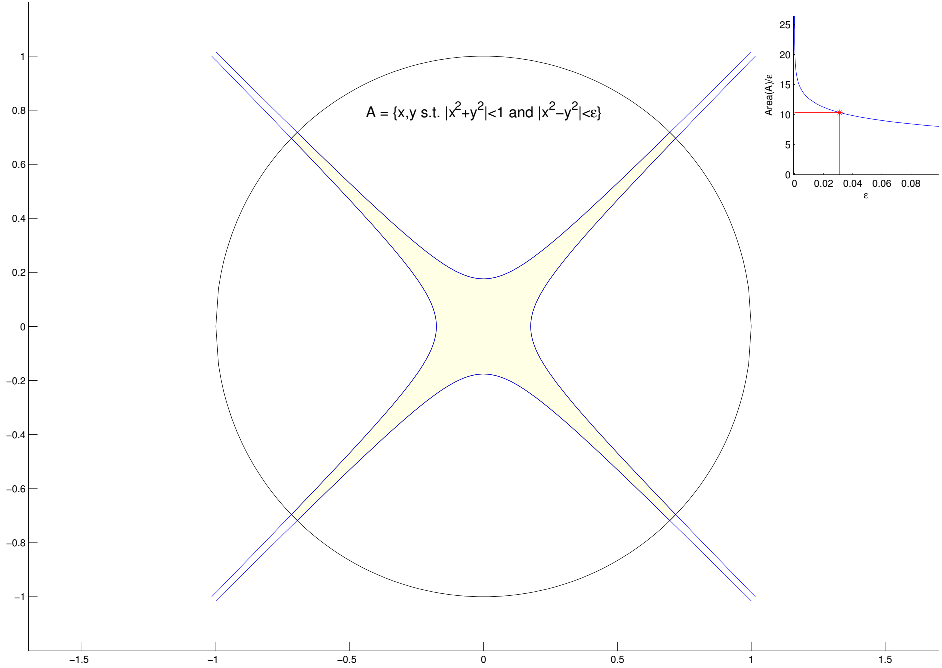
Area(A)/ε
ε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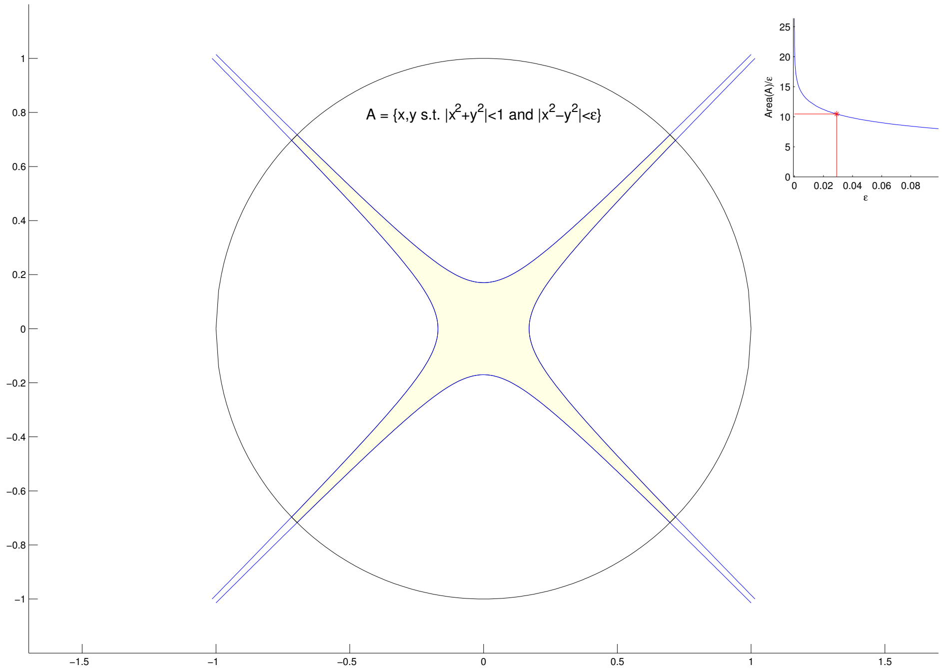

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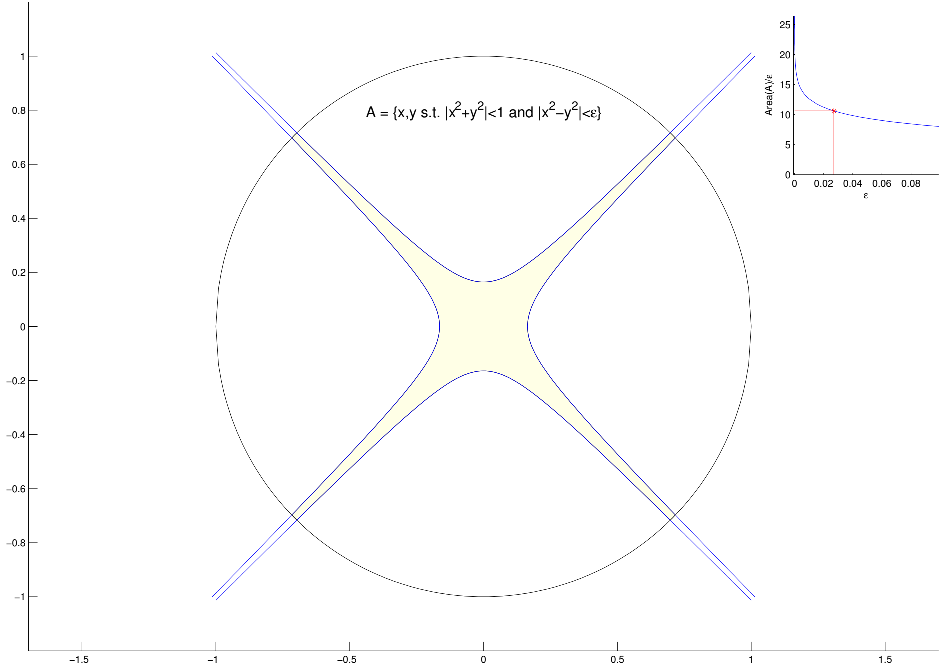
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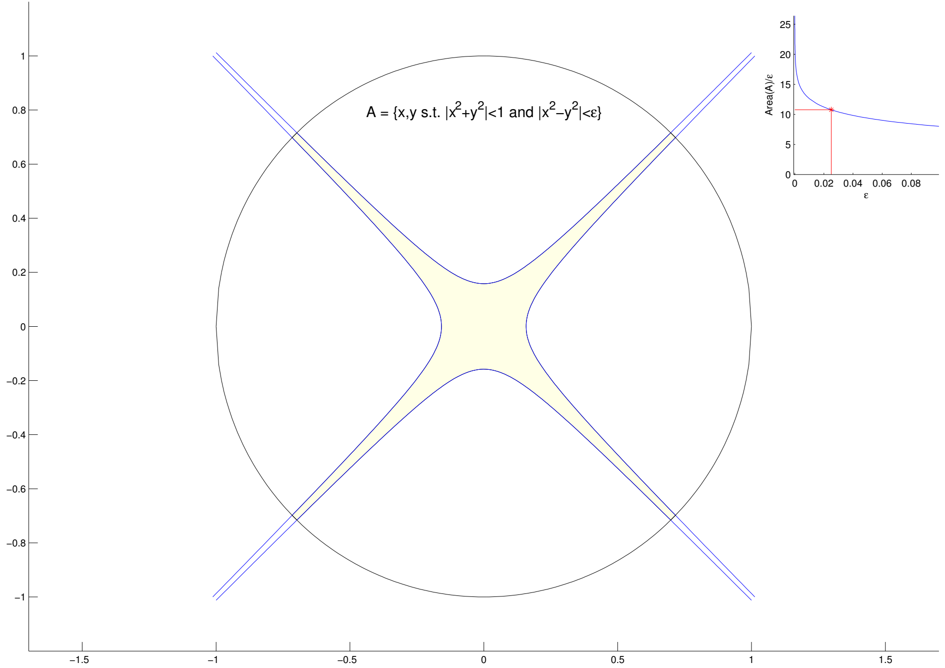
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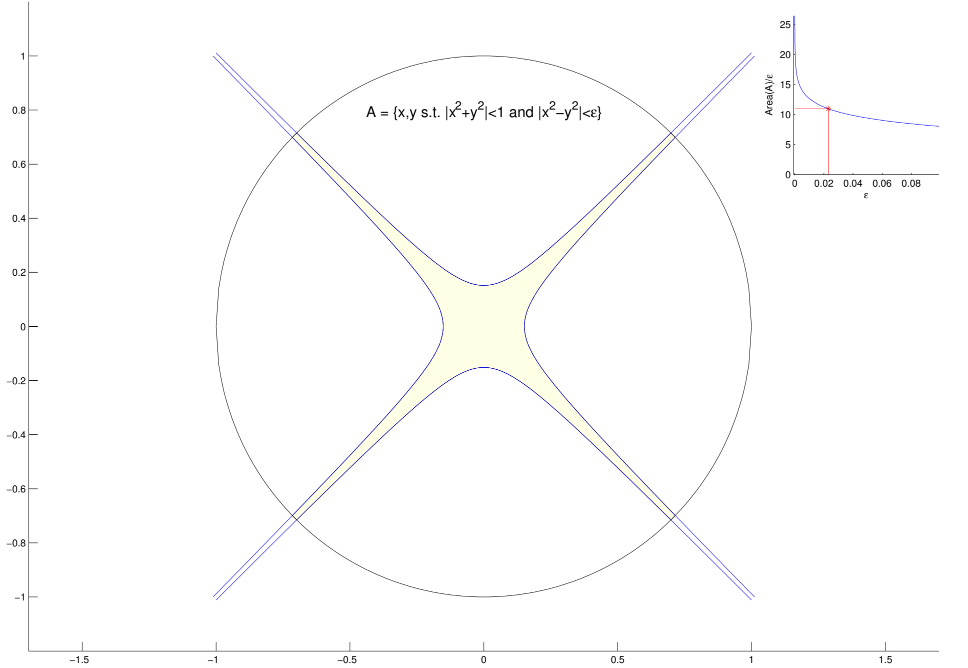
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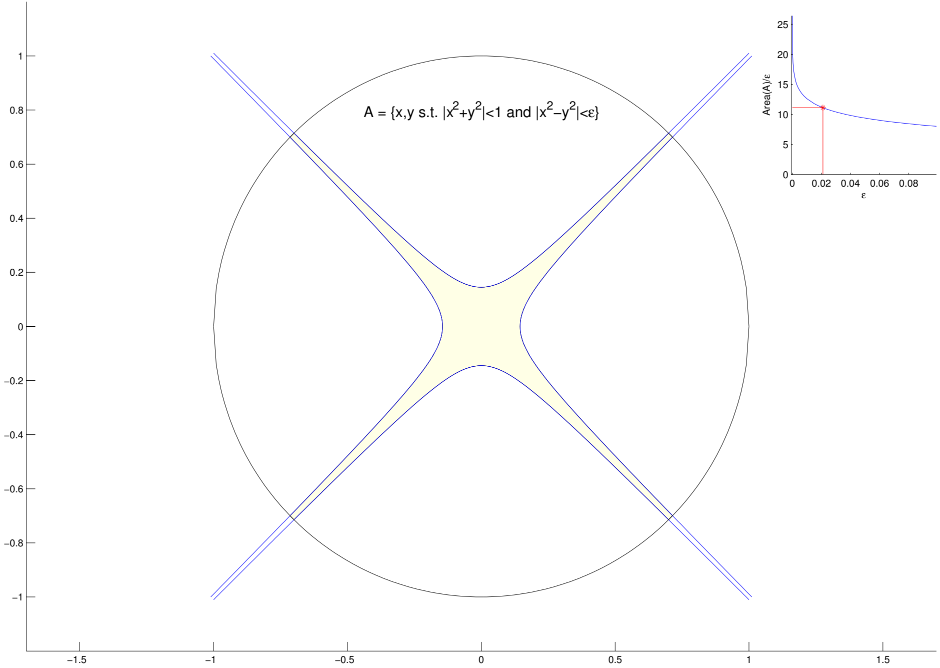

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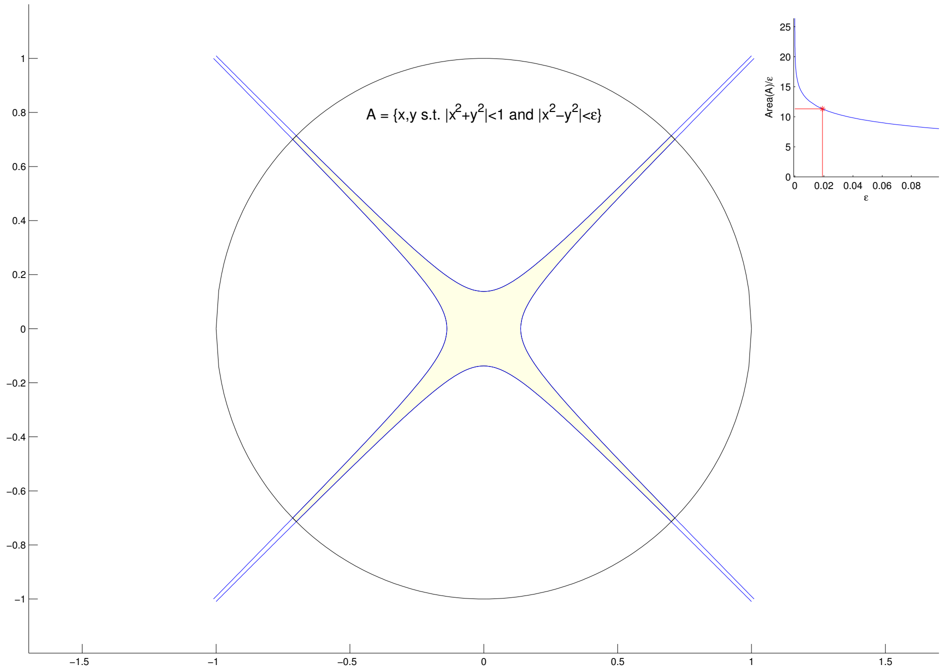

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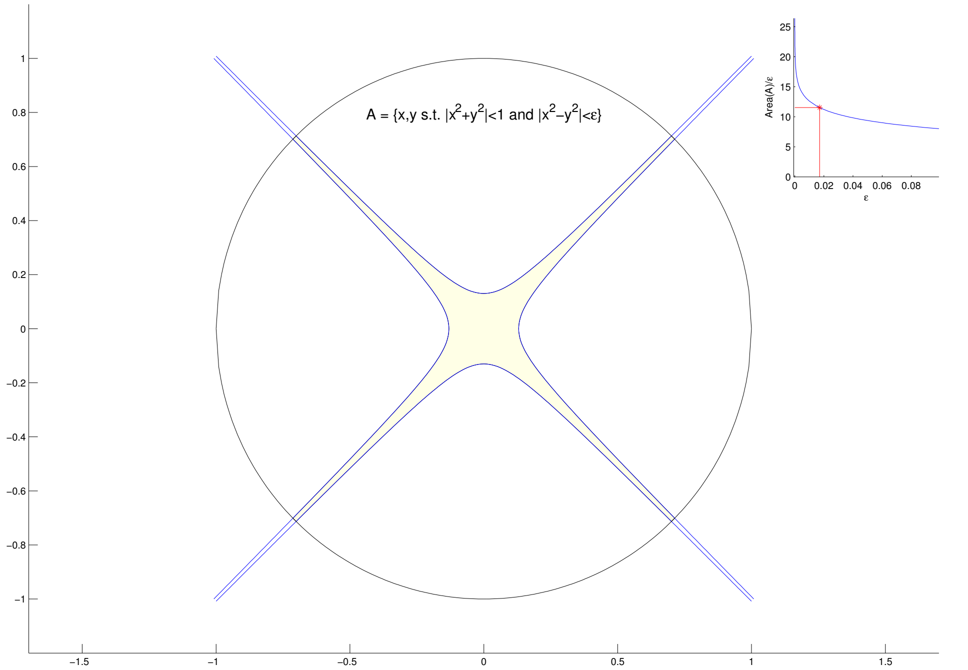
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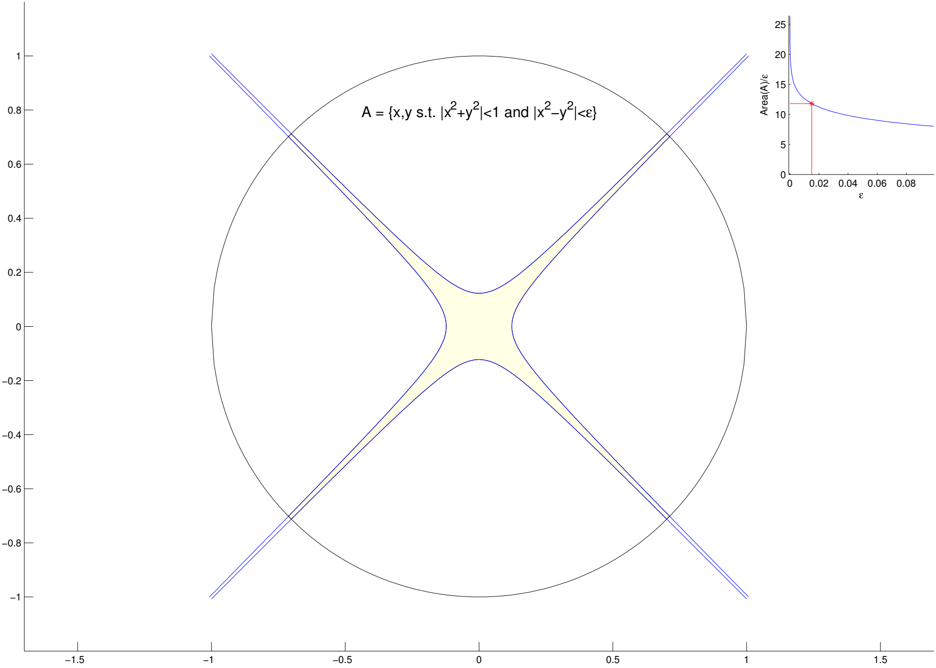

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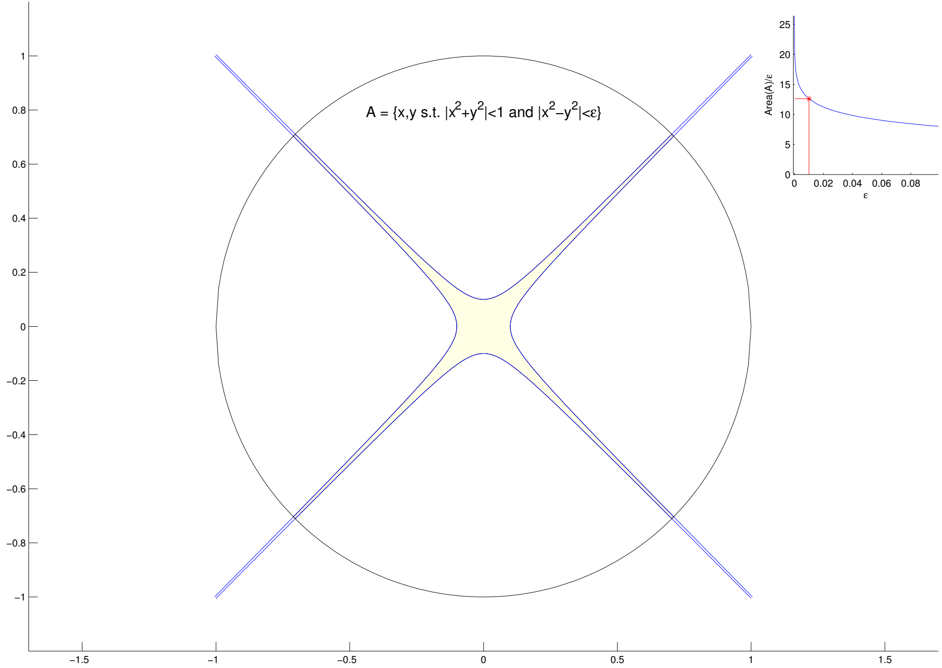
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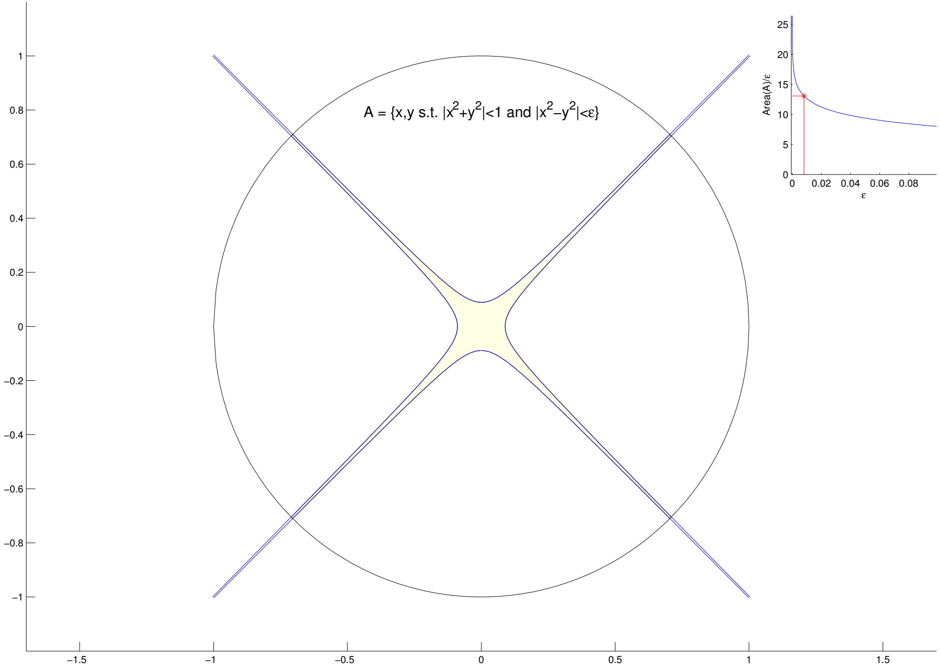

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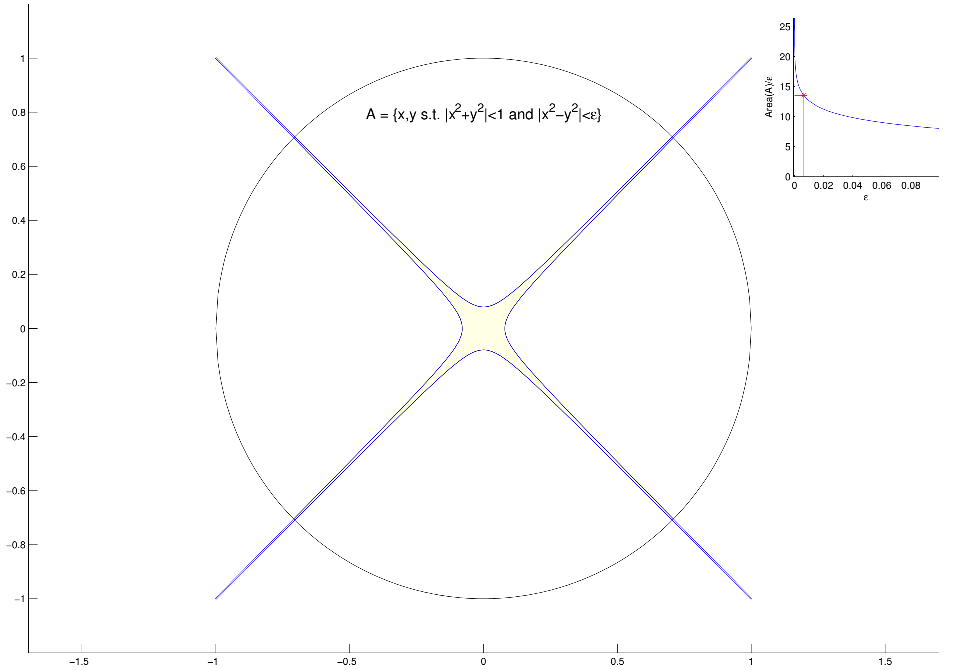

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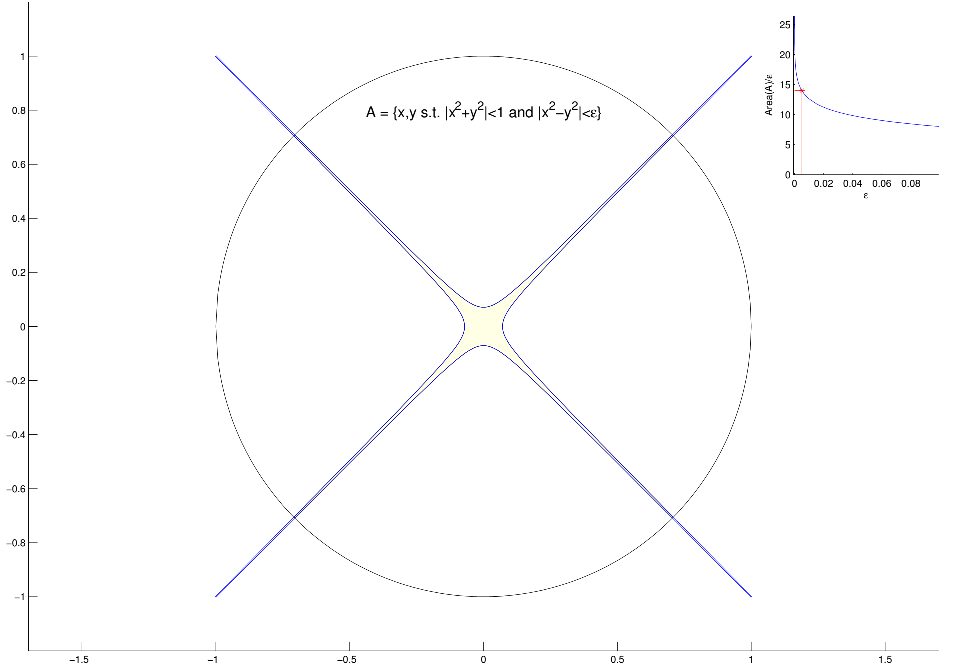
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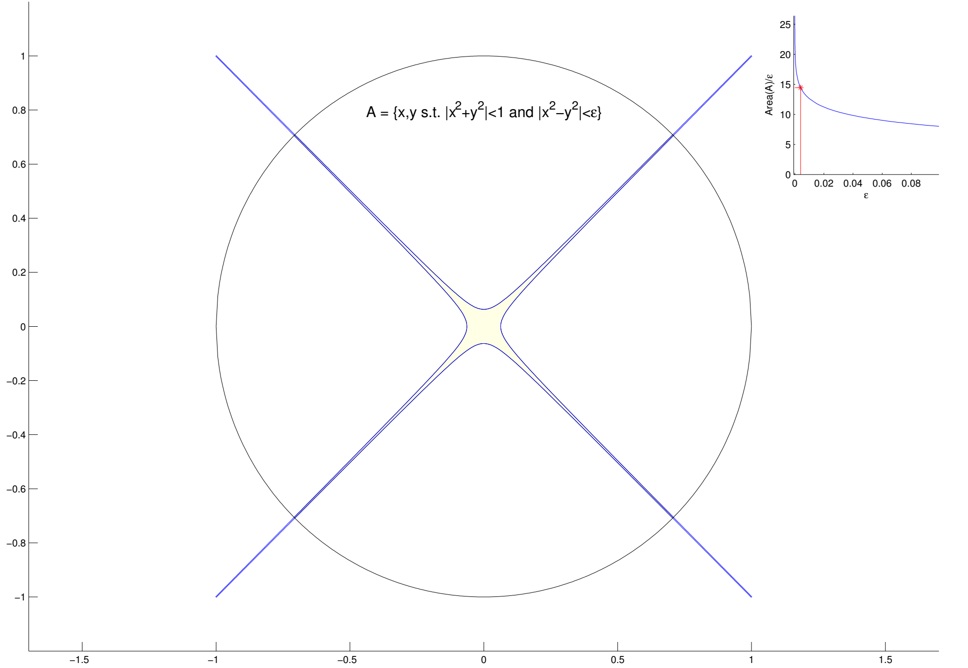

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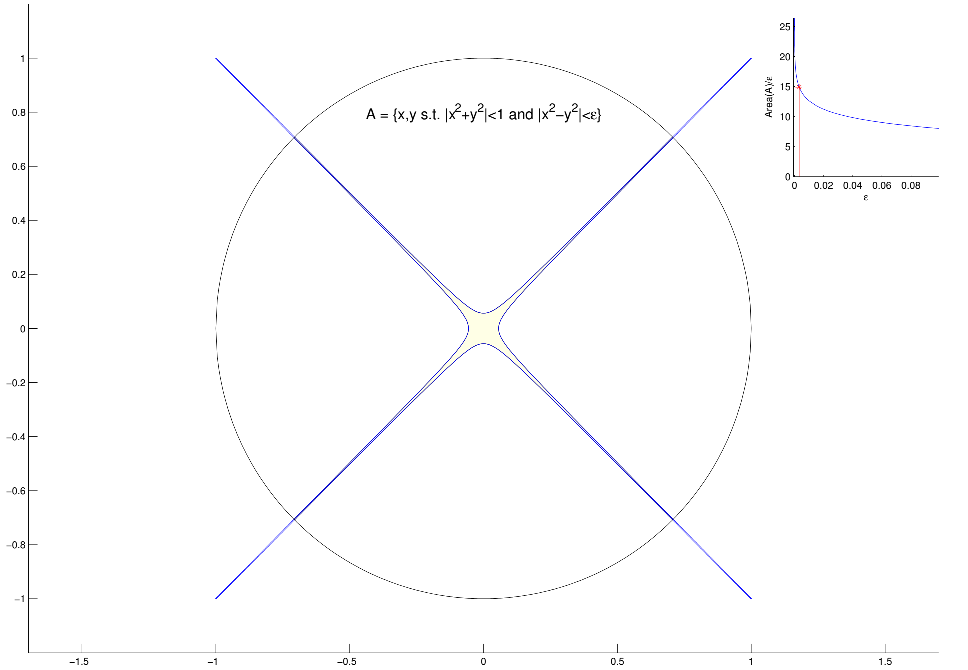
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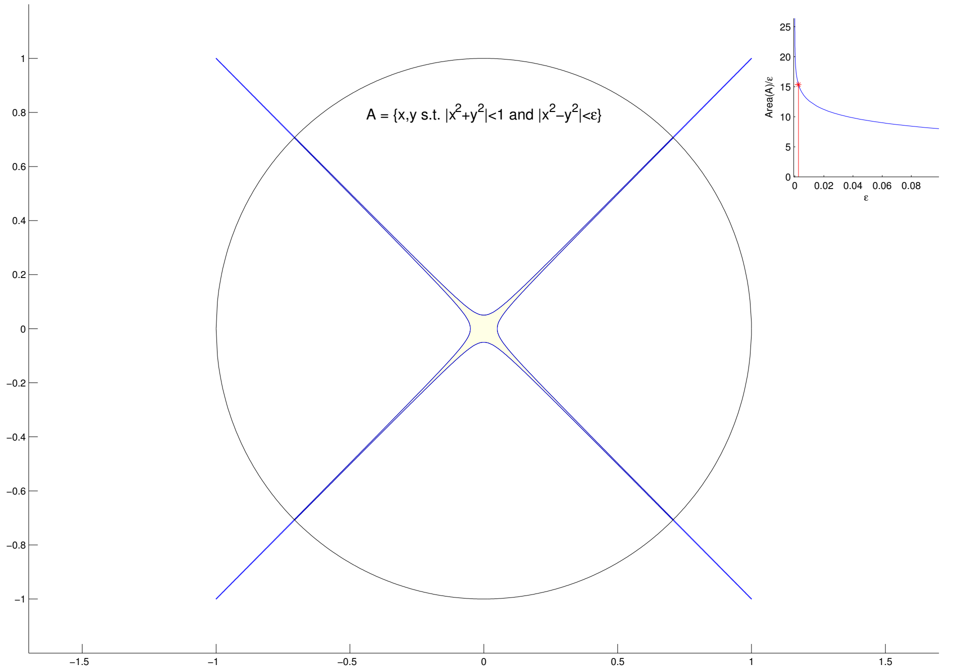

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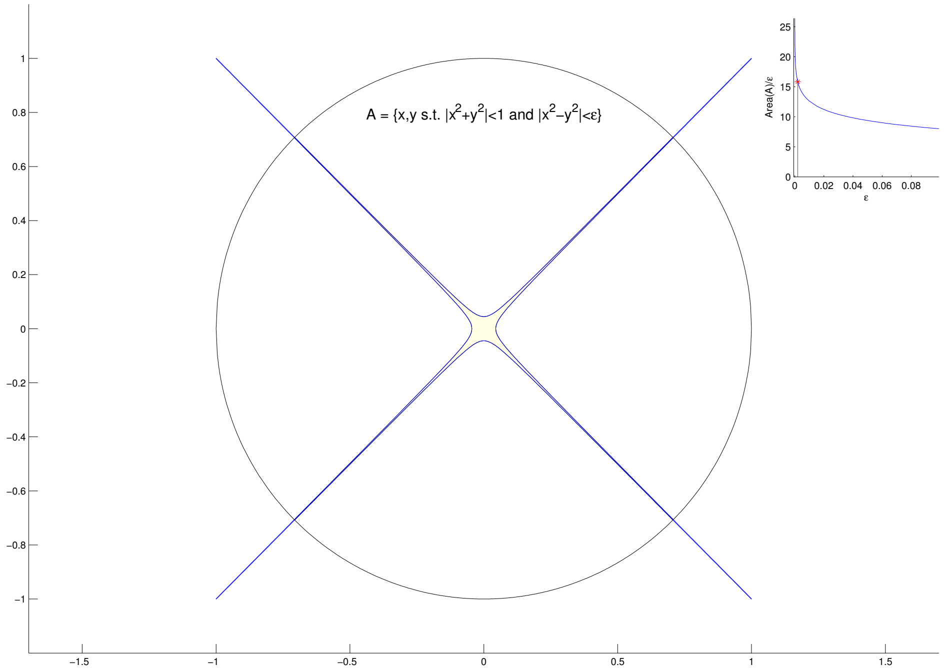
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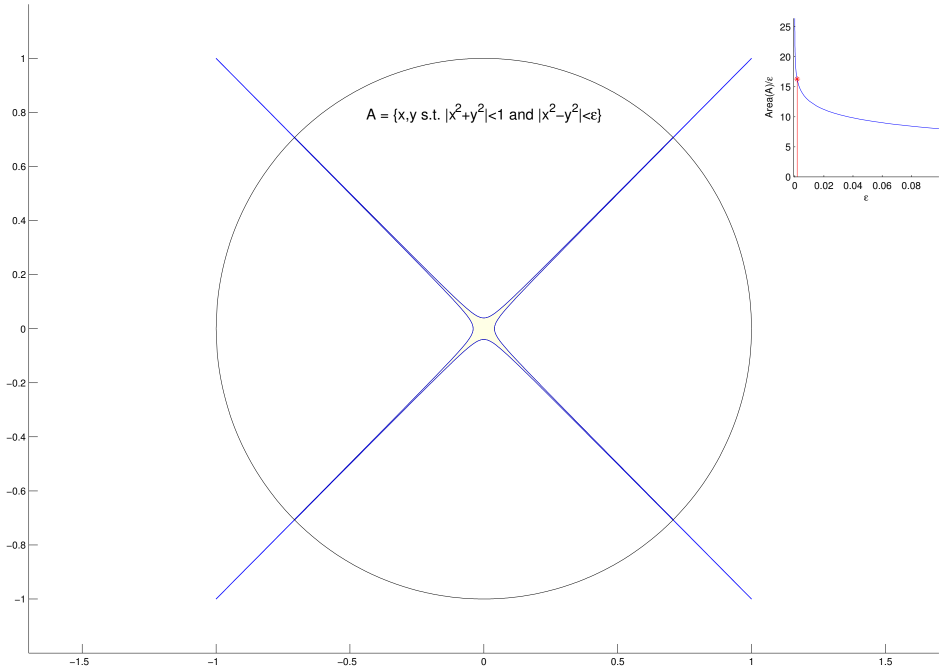
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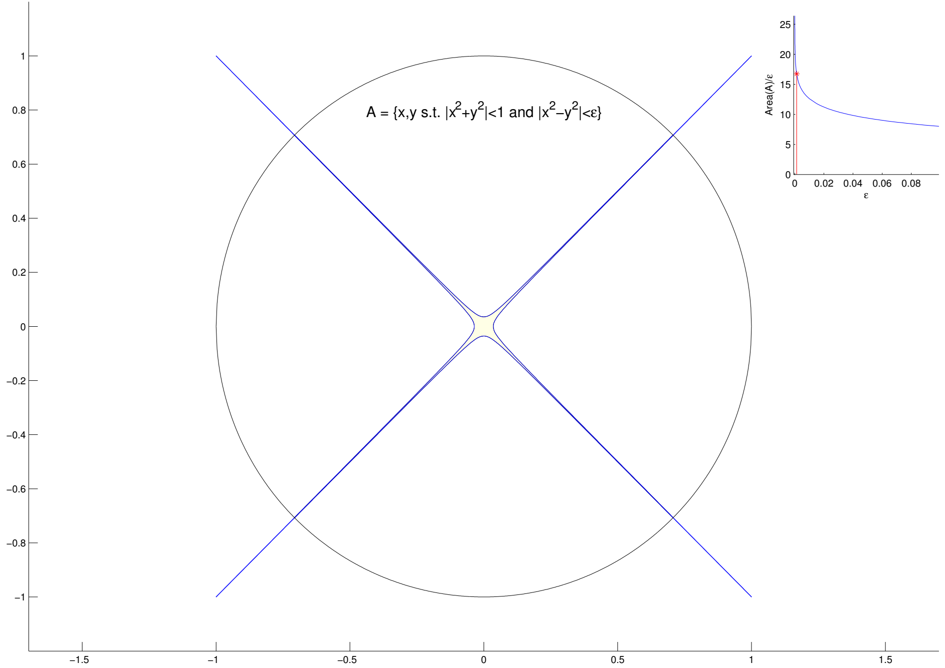
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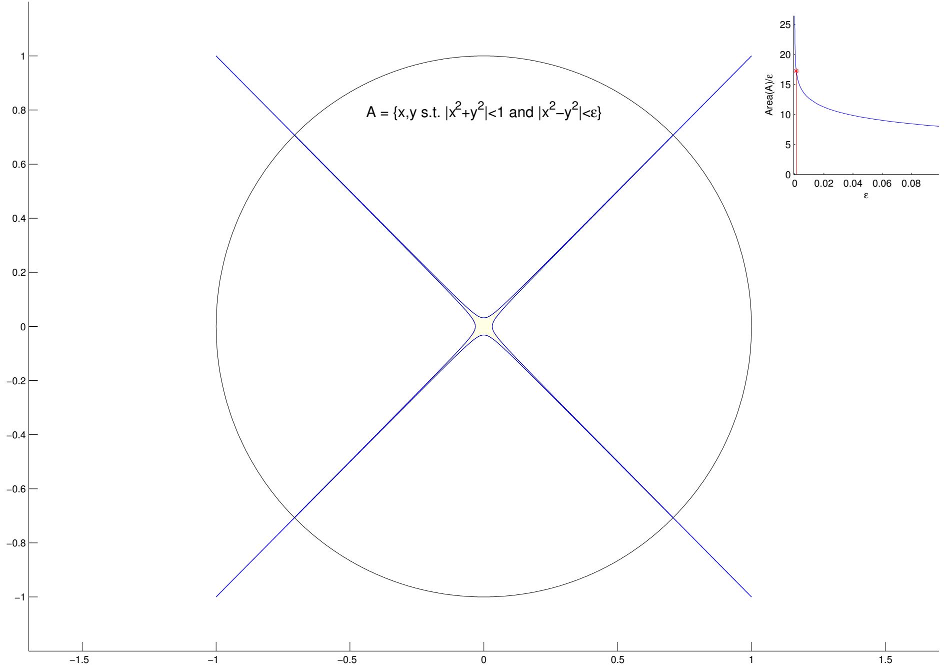
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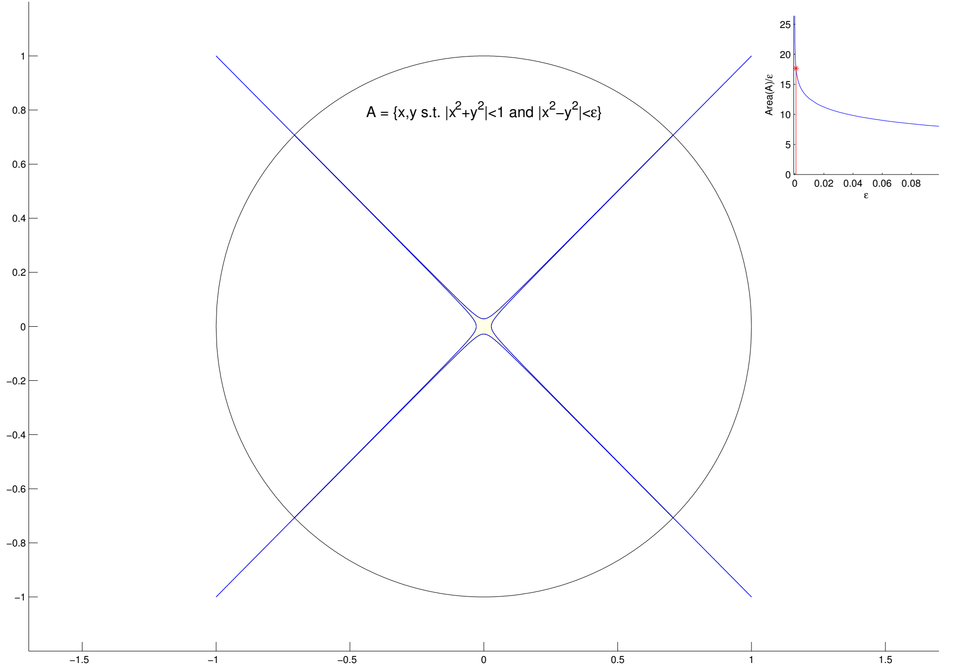
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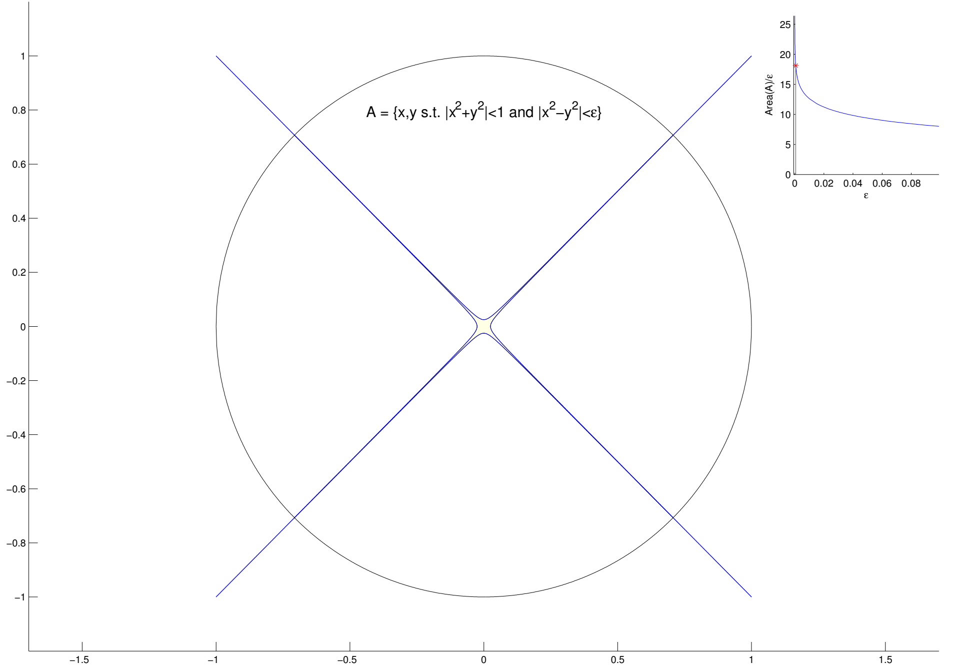
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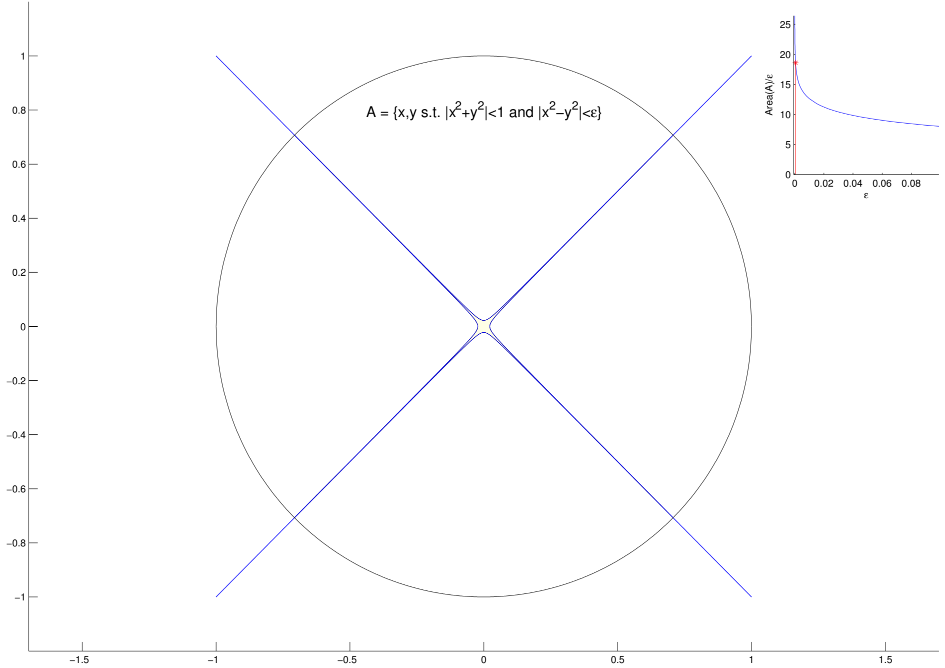

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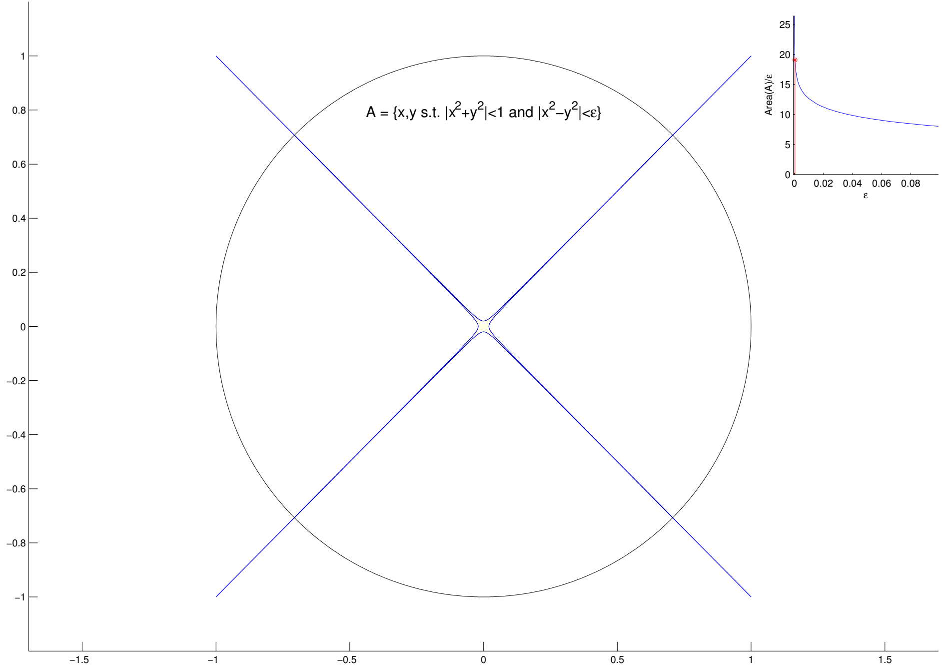

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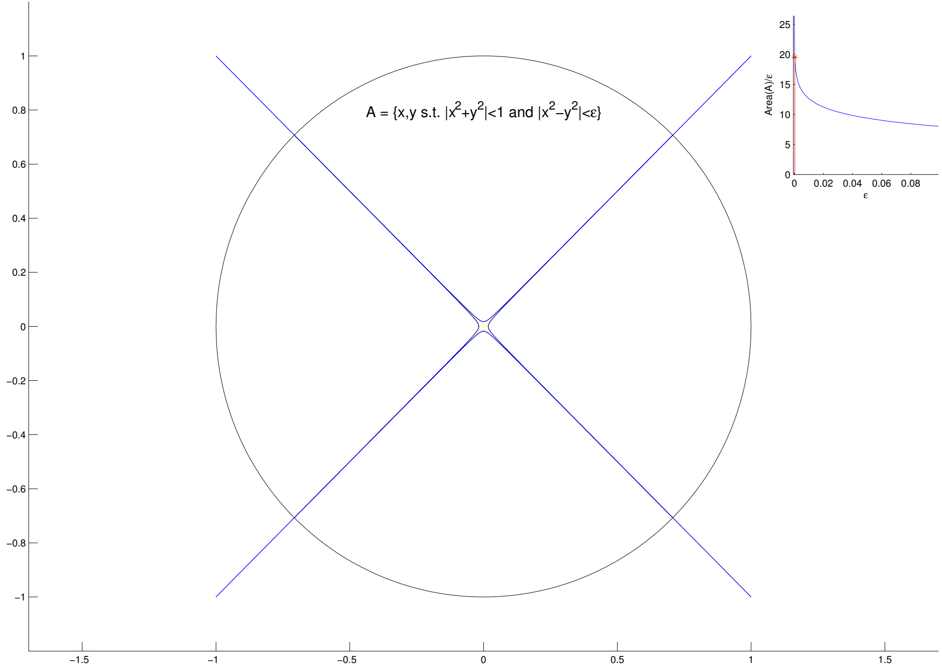
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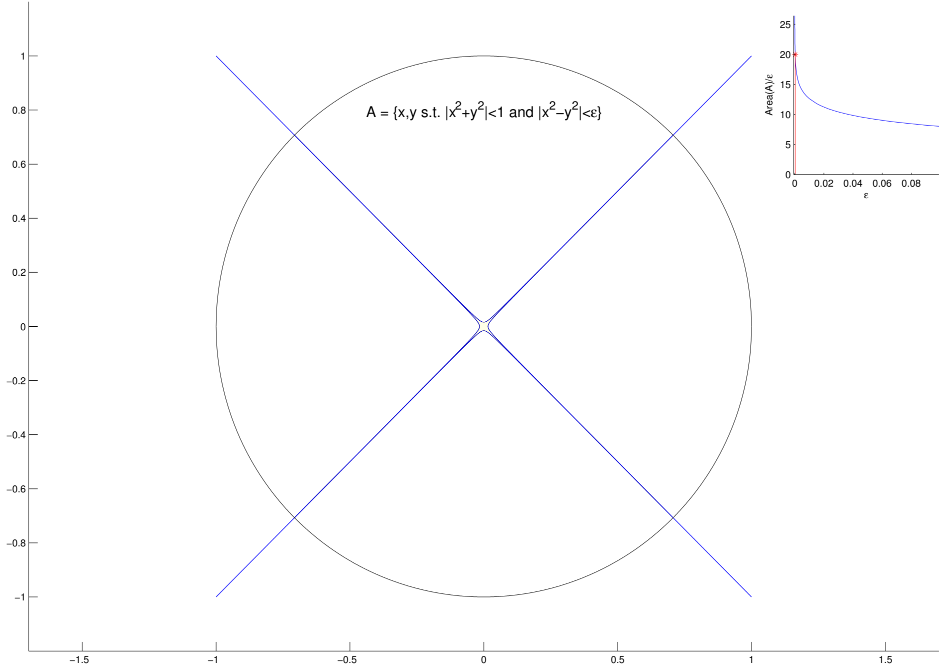
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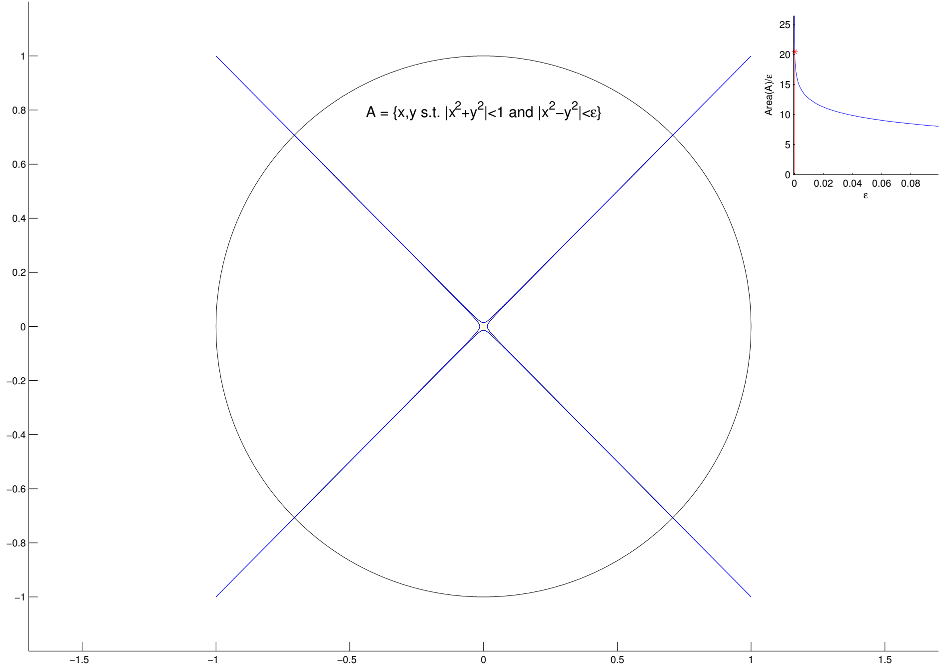
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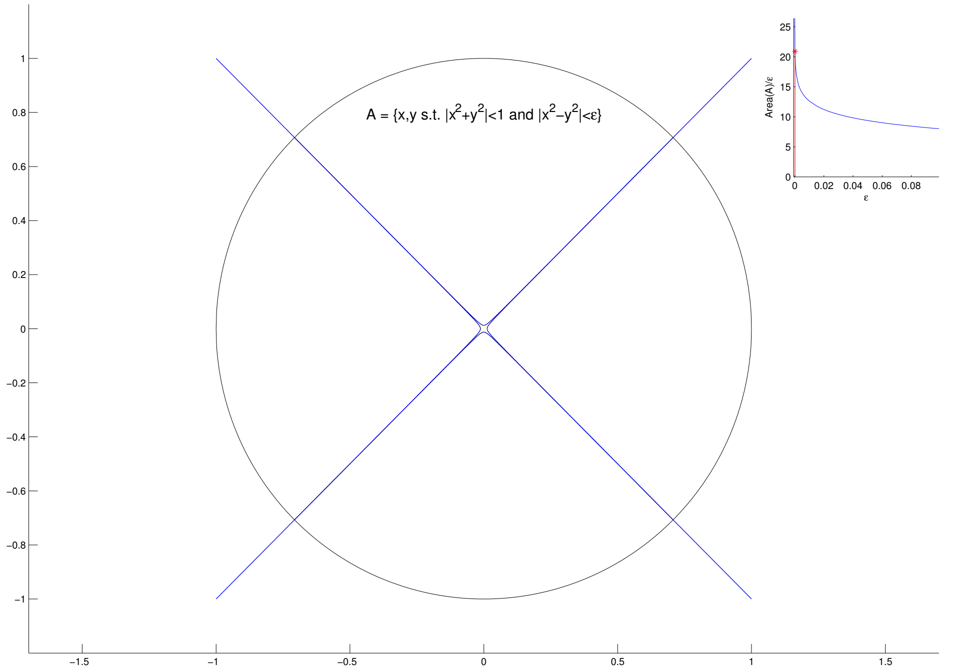

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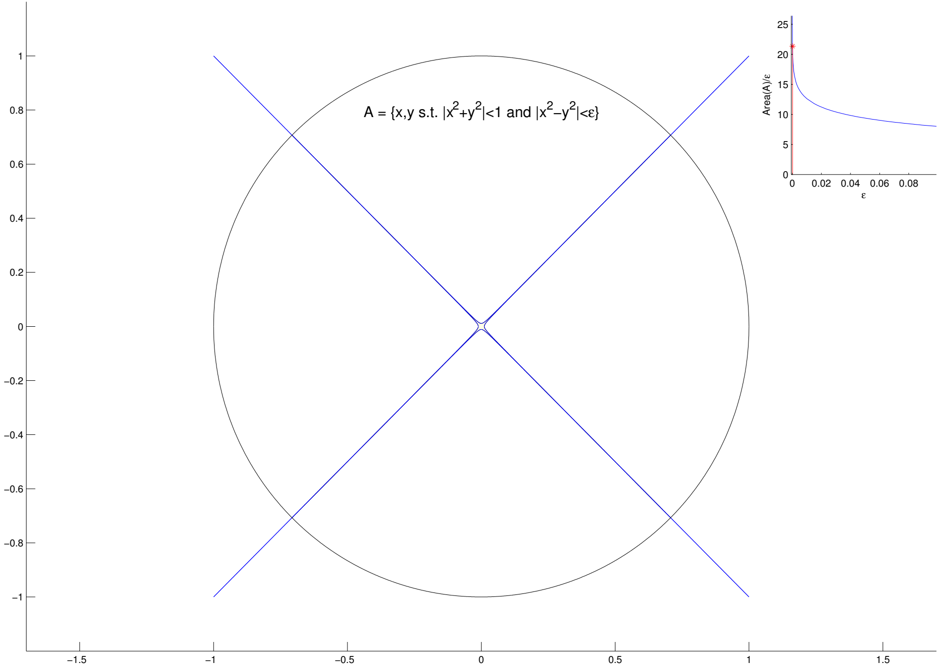

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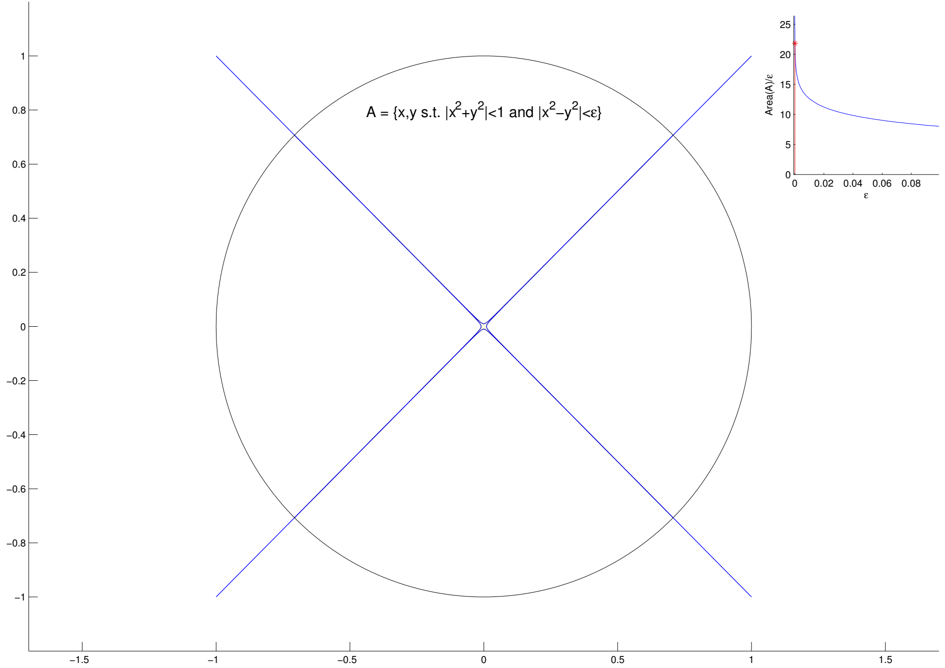

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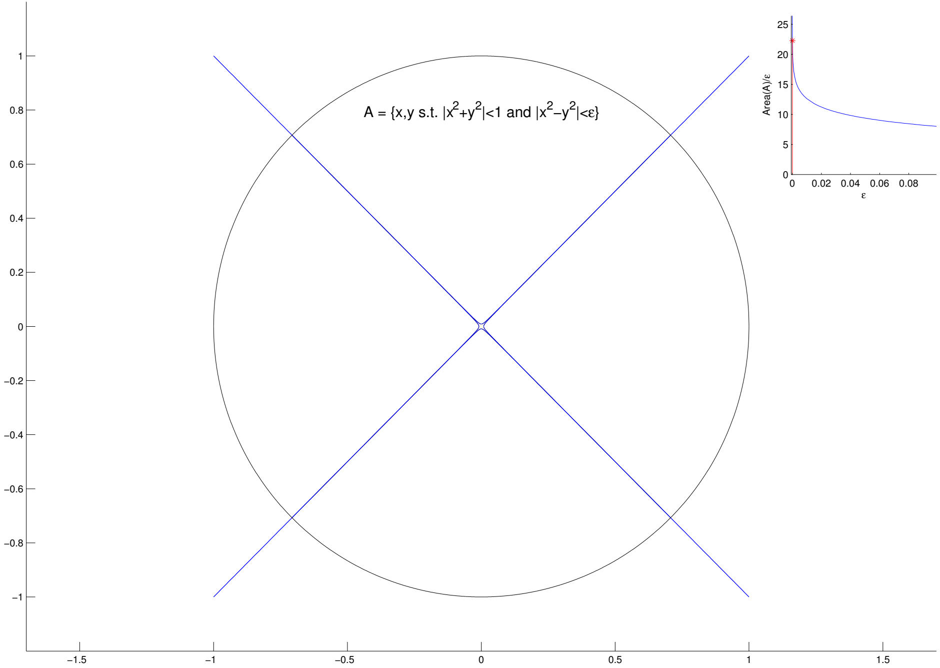

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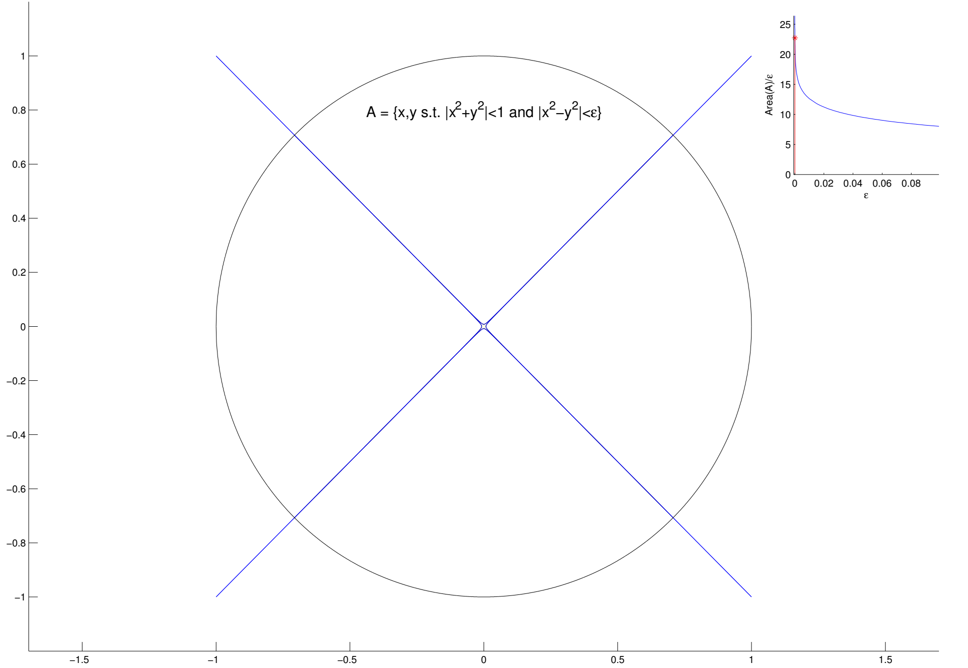
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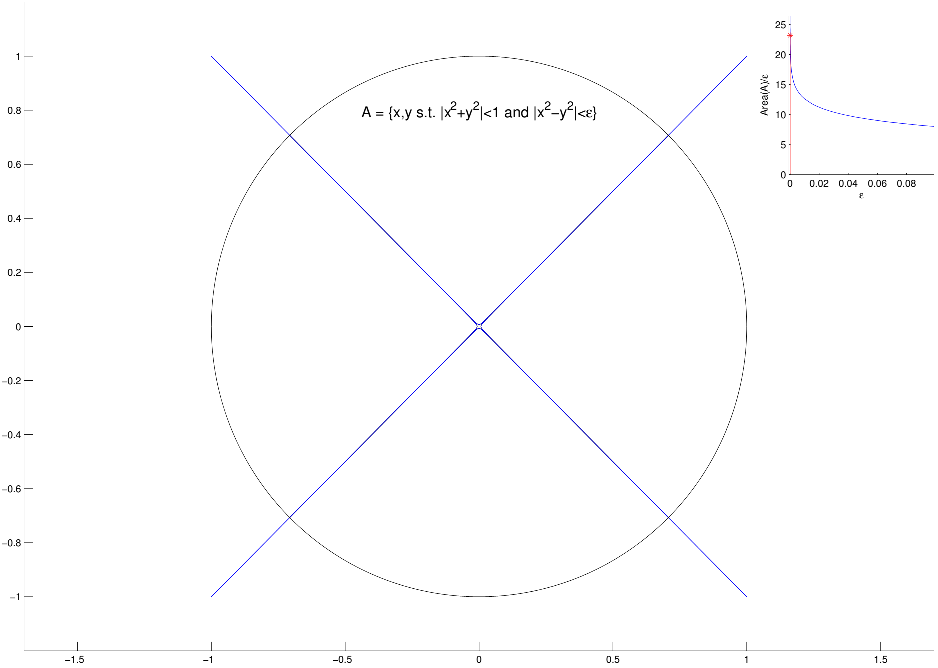
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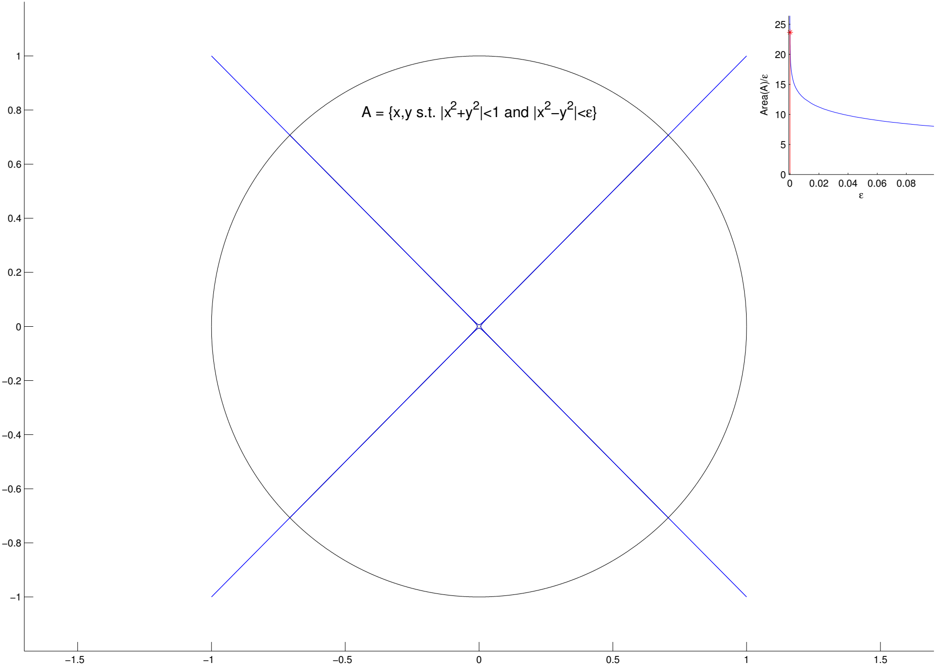
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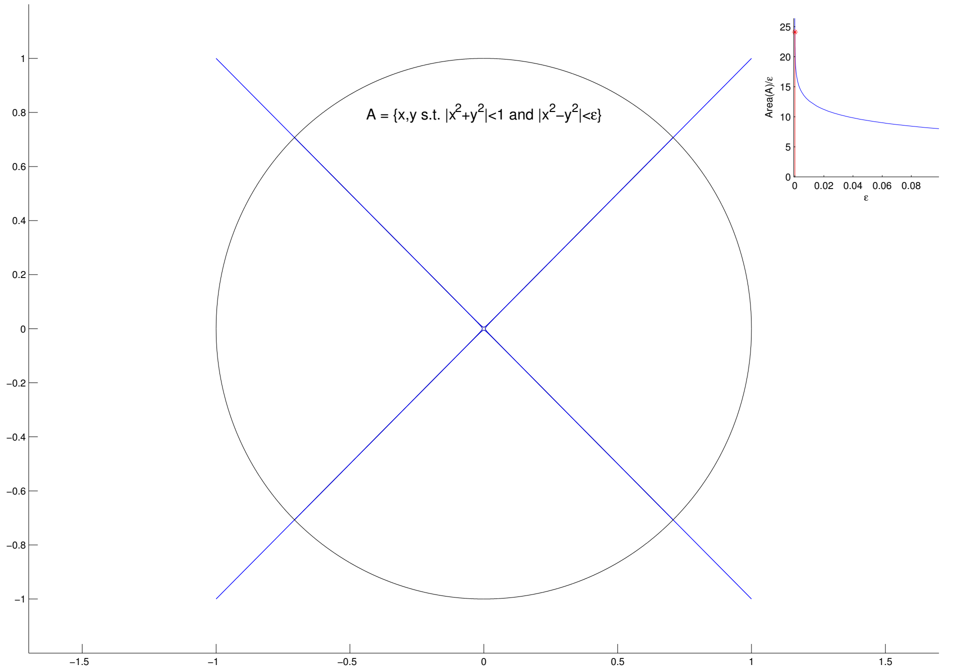

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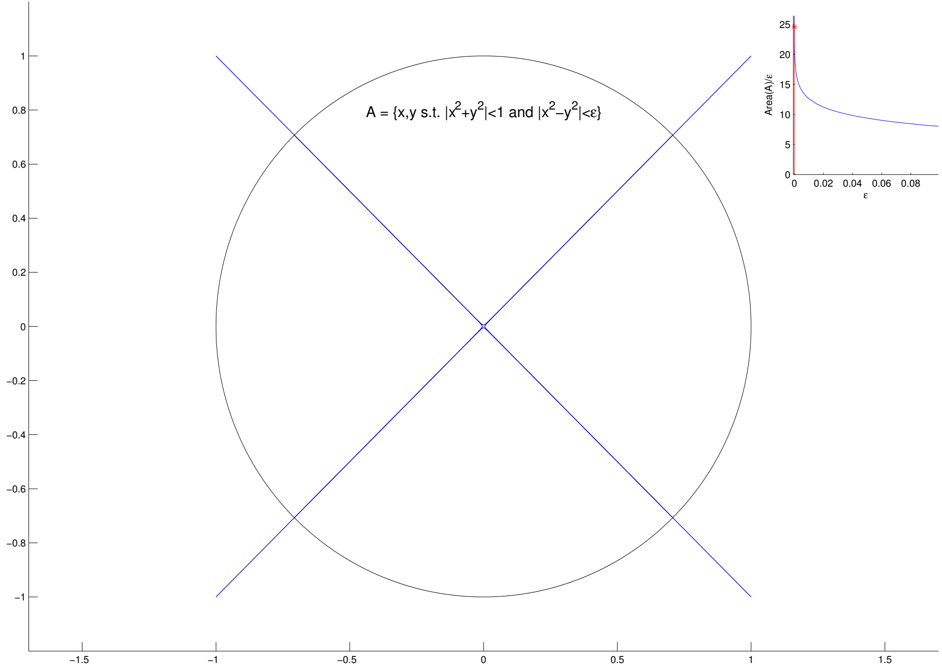
$A = \{x,y \text{ s.t. } |x^2+y^2|<1 \text{ and } |x^2-y^2|<\varepsilon\}$
Area(A)/ε
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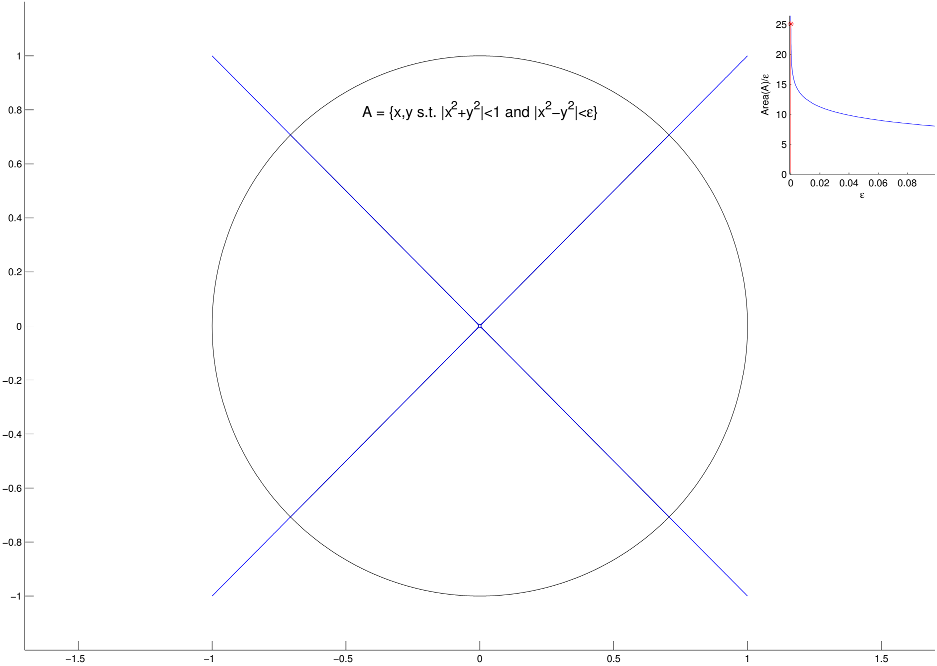
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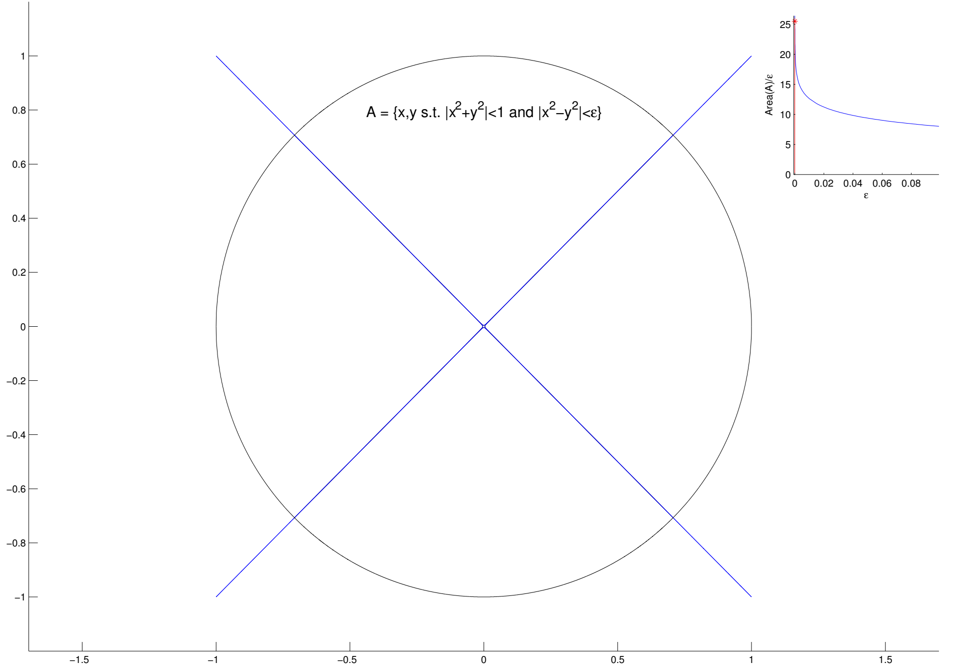
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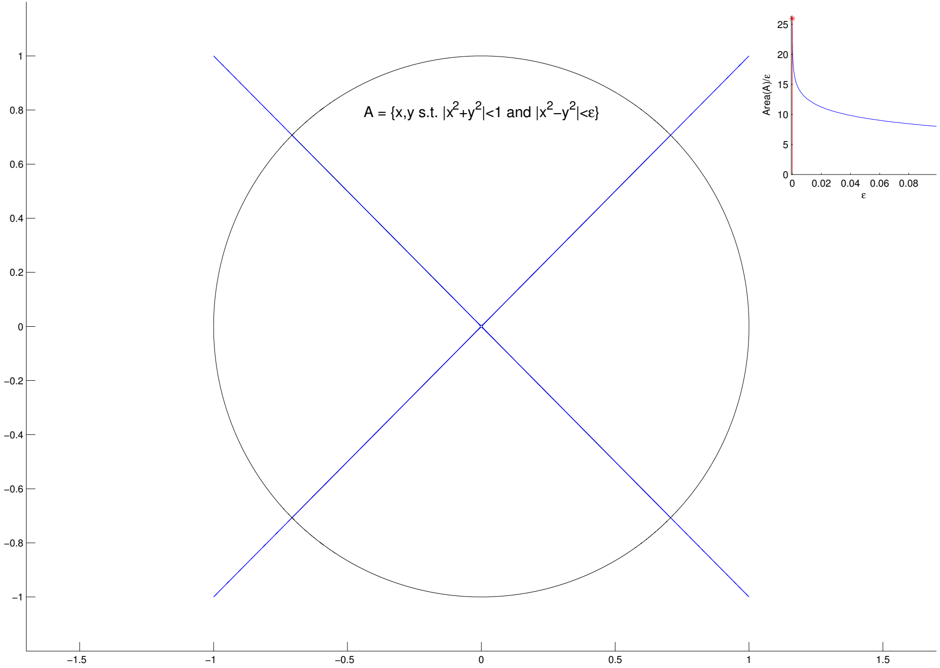
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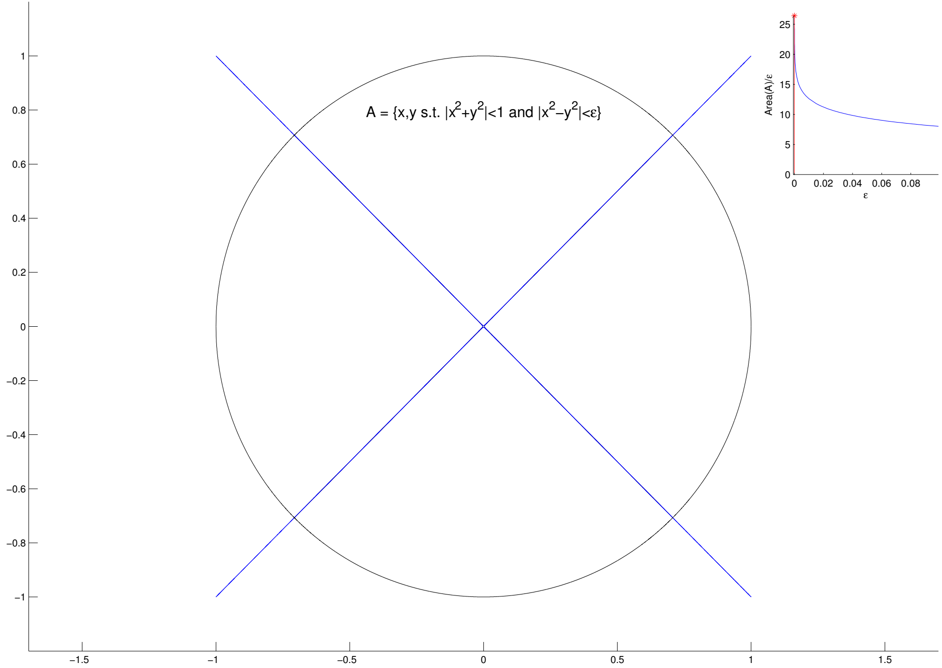

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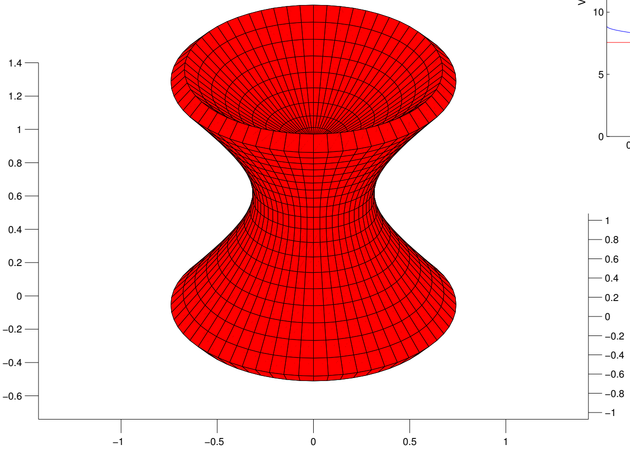
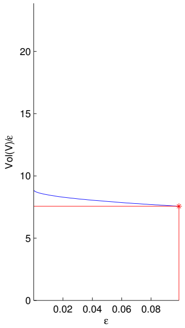
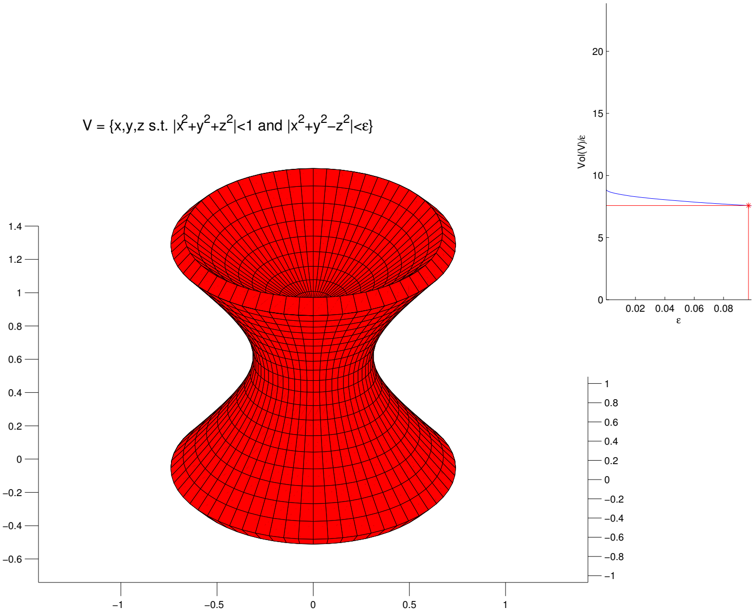
0.2
−0.2
−0.4
−0.6
−0.8
−1
−1.5
−0.5
0.5
1.5

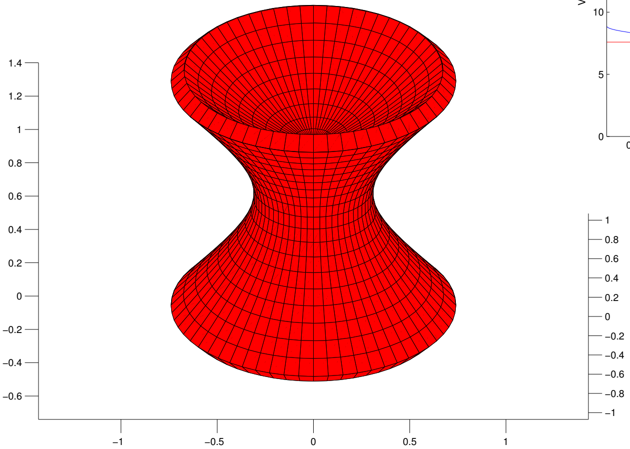
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

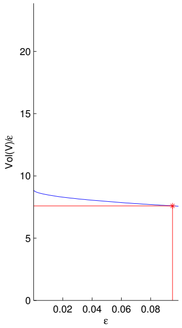




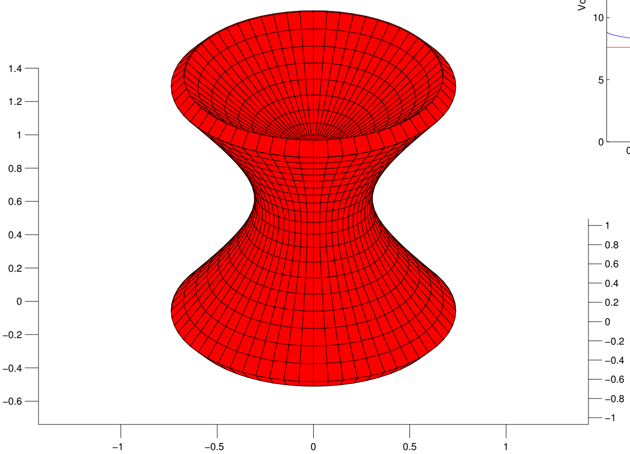

V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
20
15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−0.5
0.5

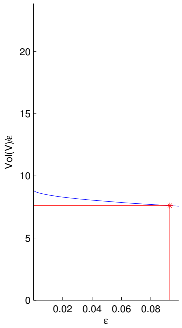
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

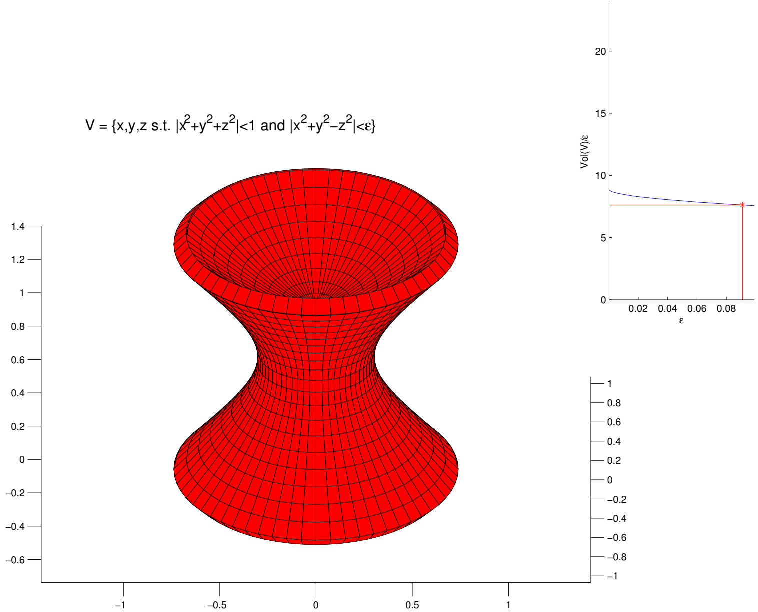




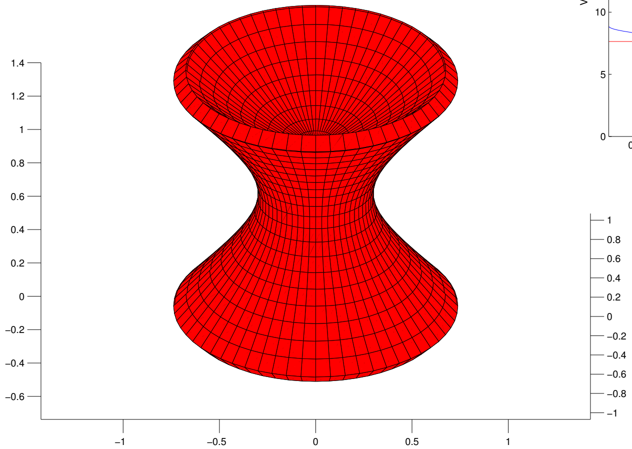
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

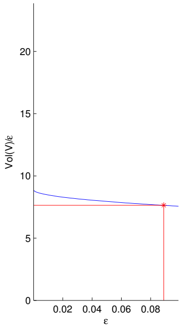


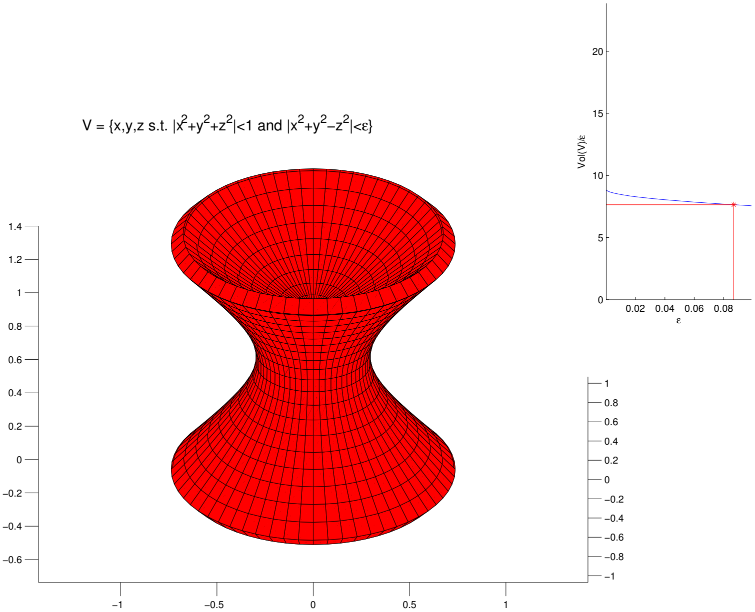



V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
20
15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
0.5
1
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

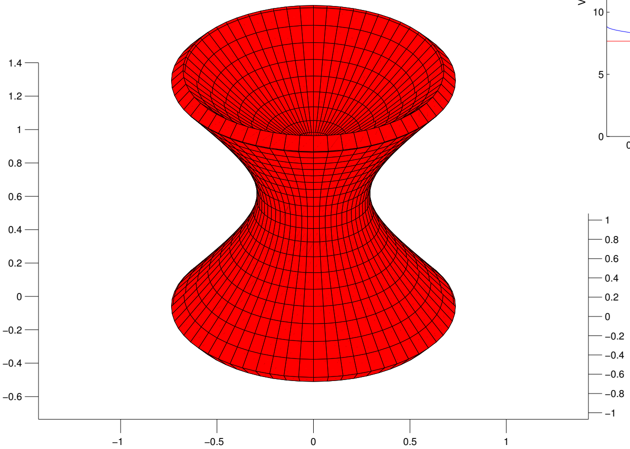
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

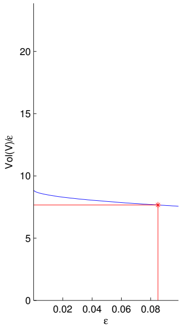


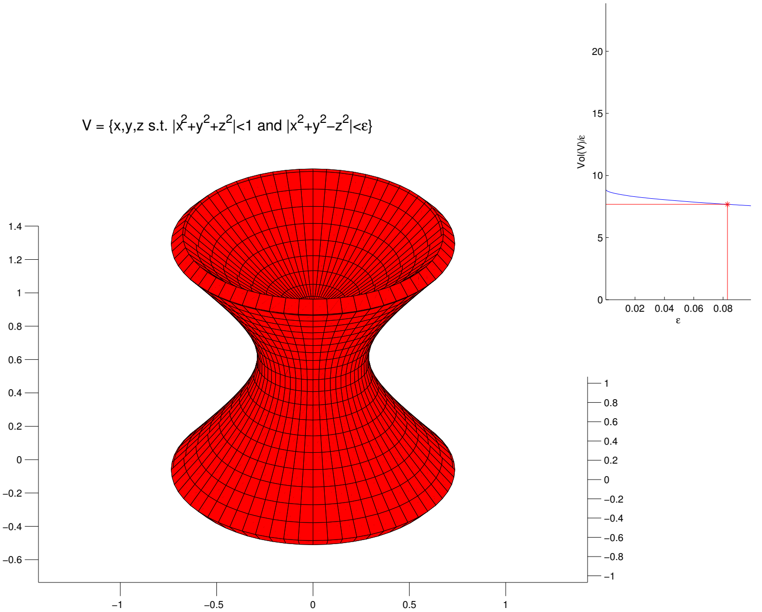


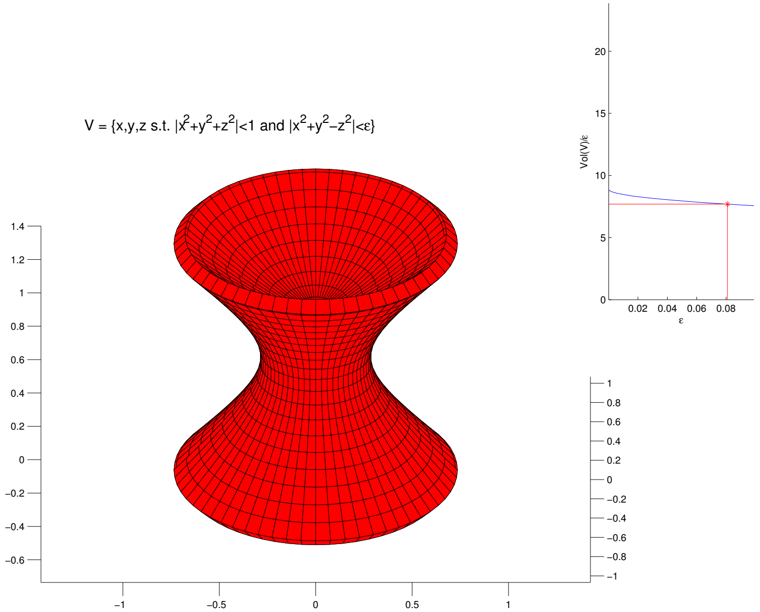

V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
0 5 10 15 20
0.02 0.04 0.06 0.08
1.4 1.2 1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6
−1 −0.5 0 0.5 1
1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6 −0.8 −1

$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

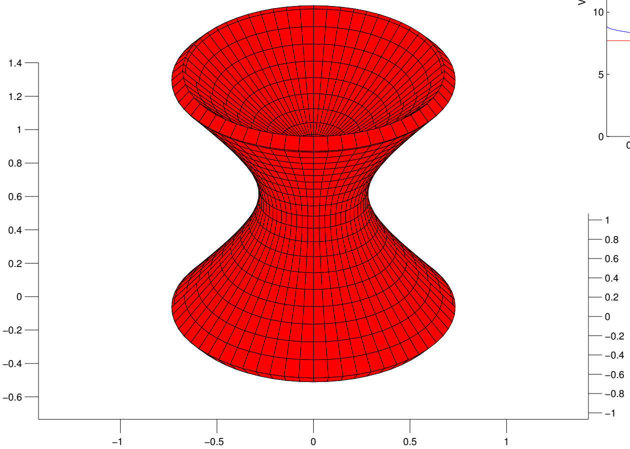


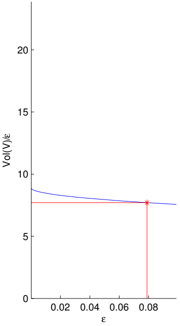



$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$
Vol(V)/ε
ε

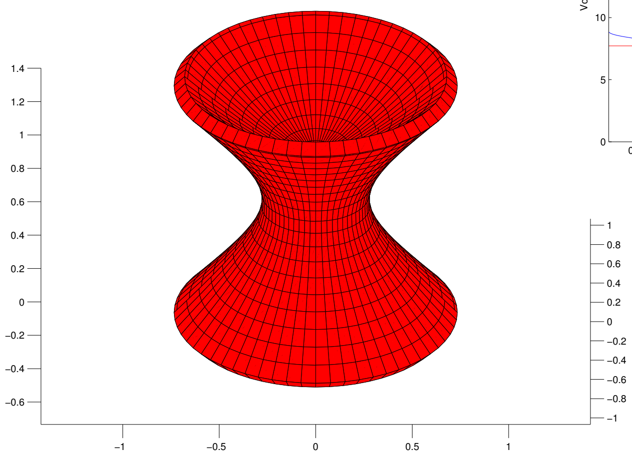

V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
0.02 0.04 0.06 0.08
0 5 10 15 20
1.4 1.2 1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6
−1 −0.5 0 0.5 1
1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6 −0.8 −1

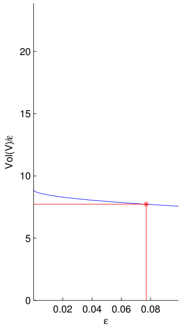
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



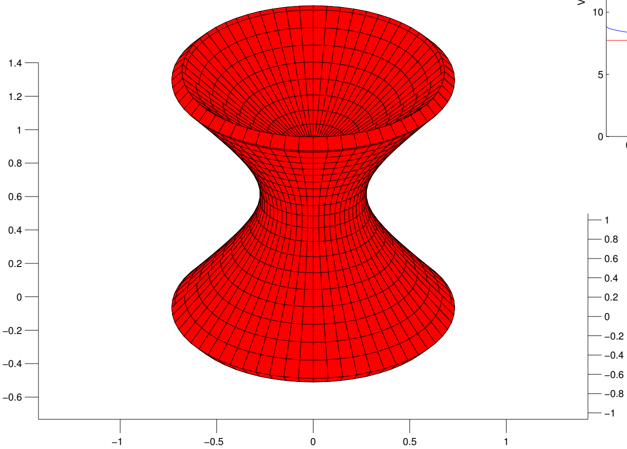


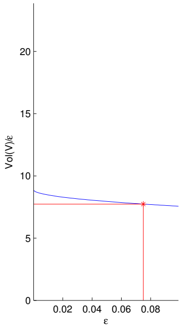
V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}



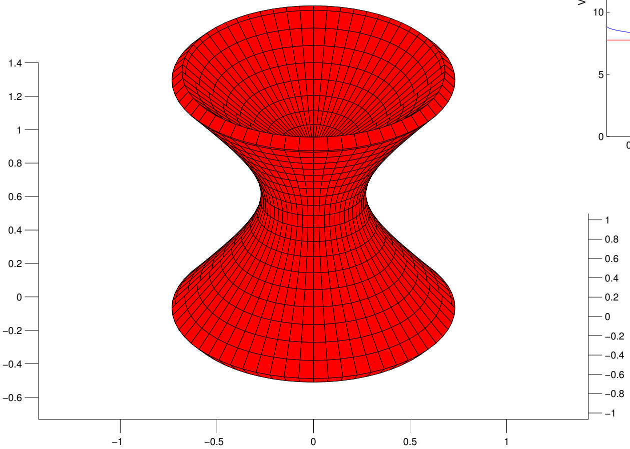


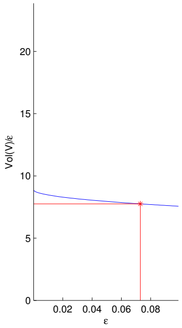
V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}



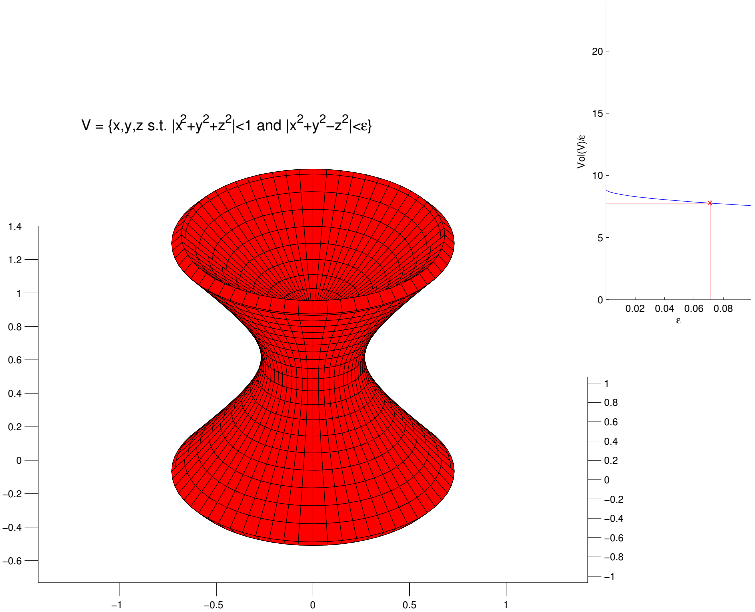


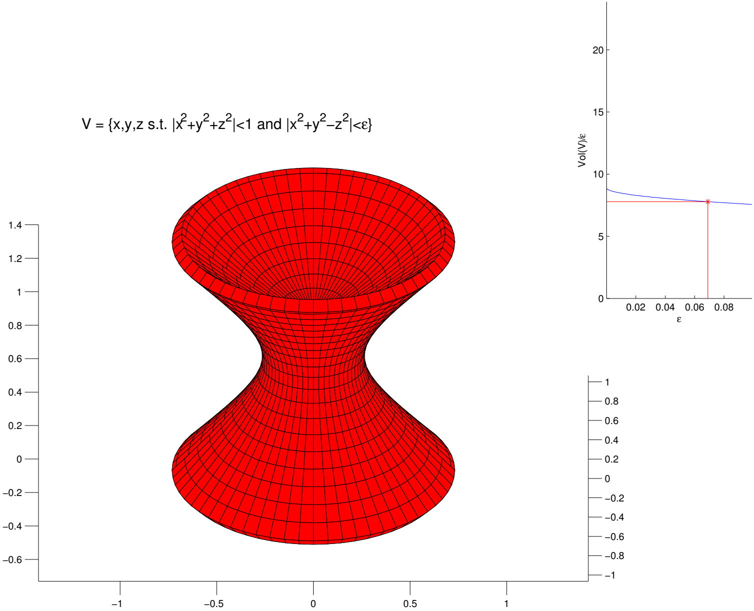
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



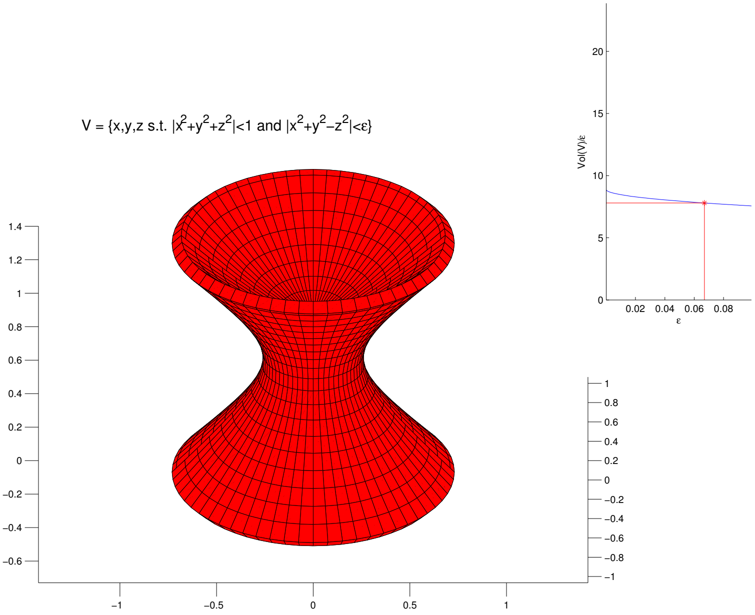


$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

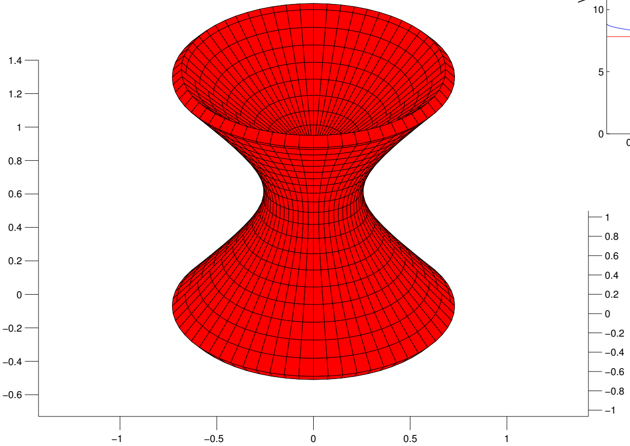


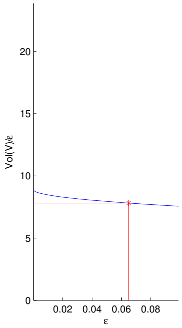

V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
0.5
1
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
20
15
10
5
0
Vol(V)/ε
0.02
0.04
0.06
0.08
ε

$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

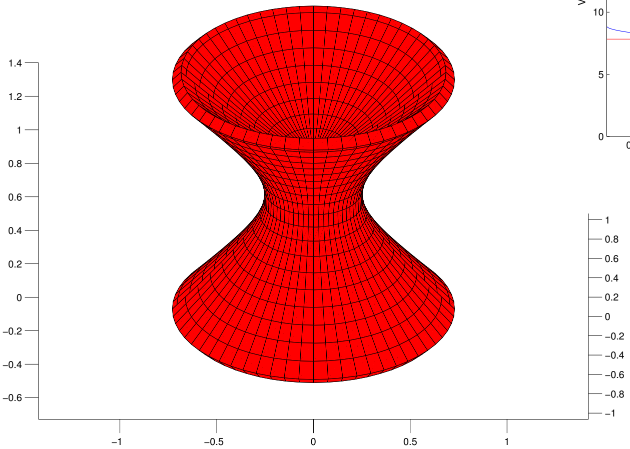


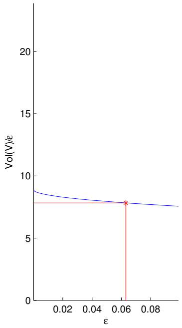
V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}



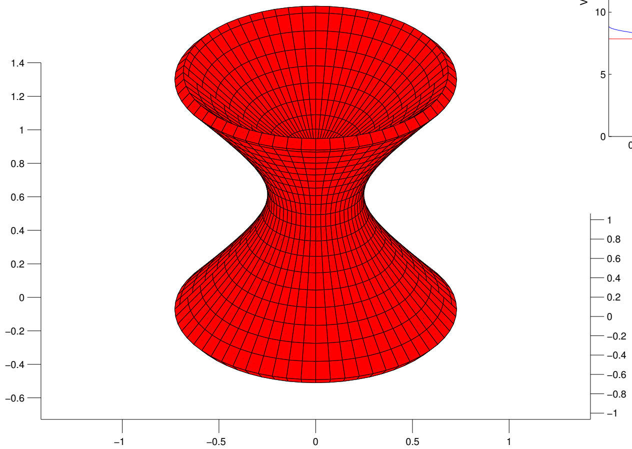


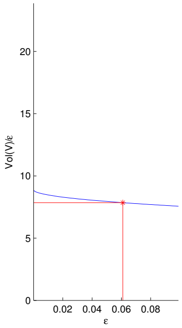
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



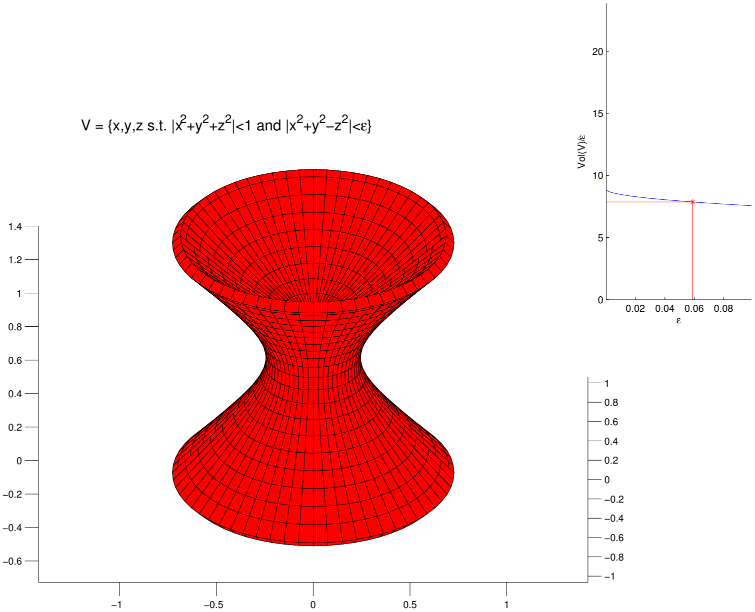


V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}

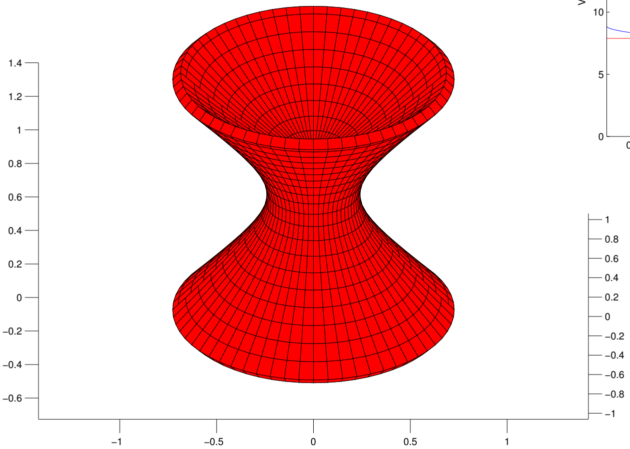


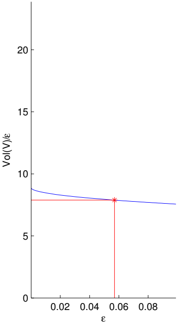


$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

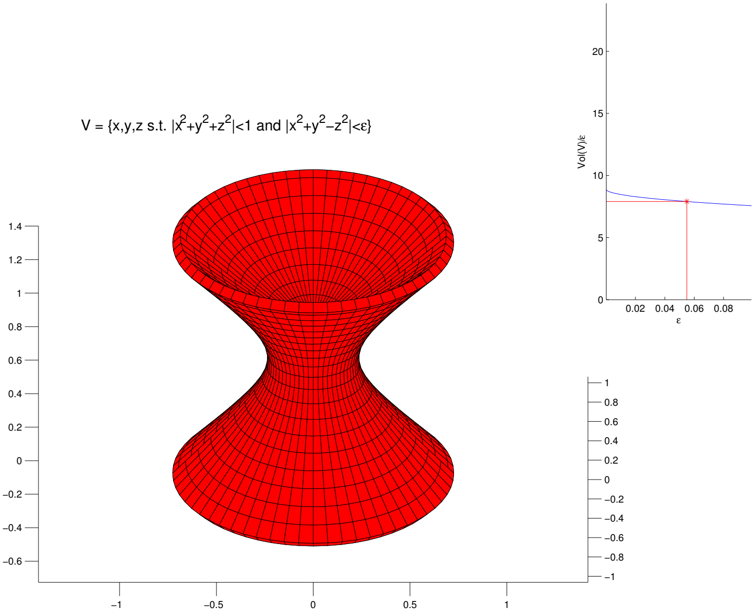


$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

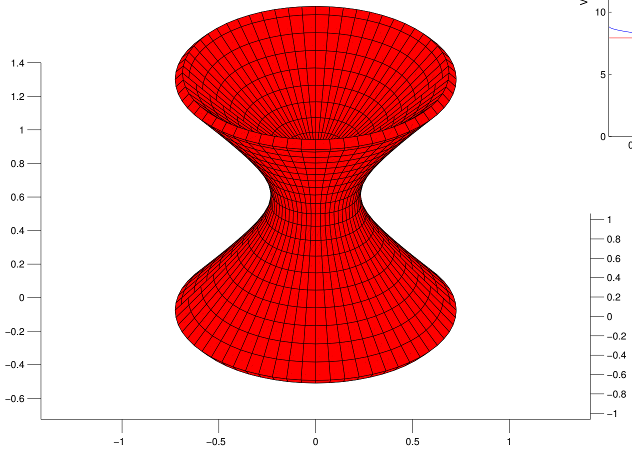


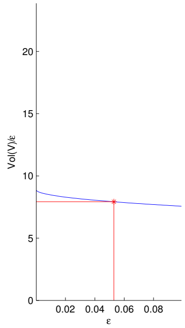


$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

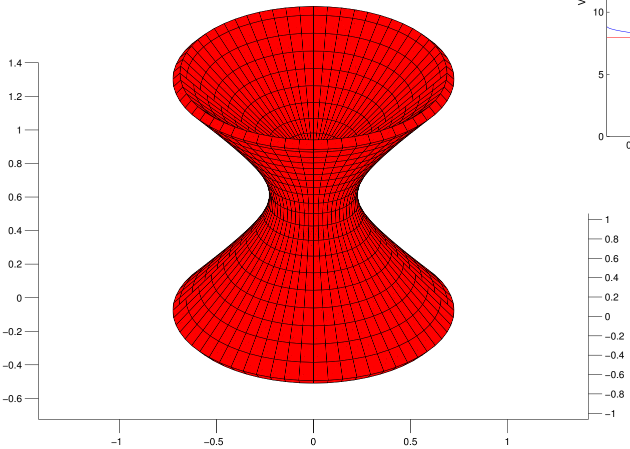


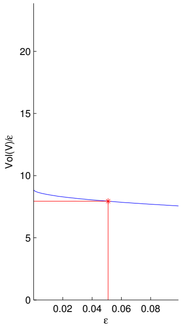
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



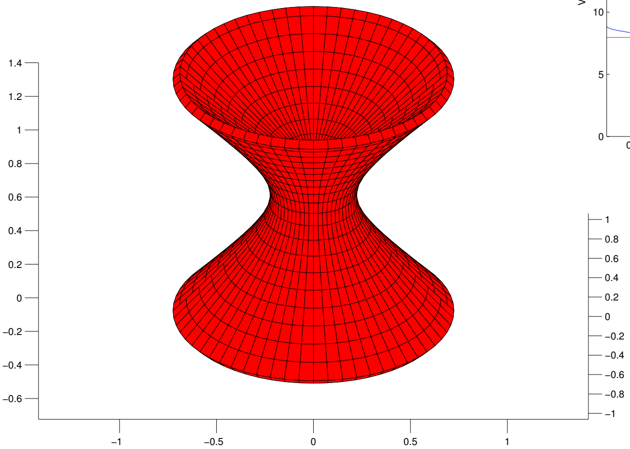


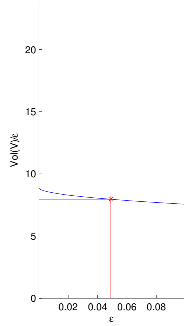
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



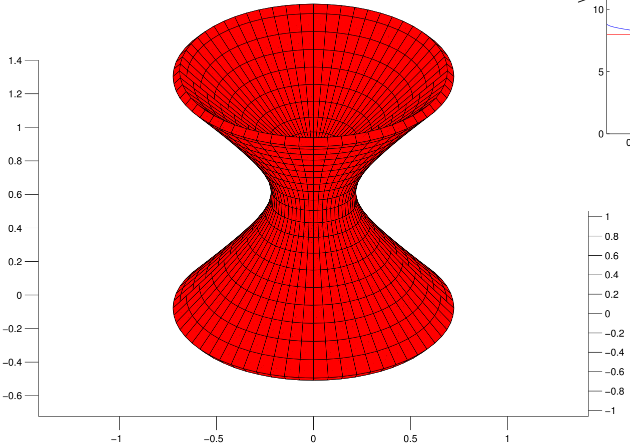


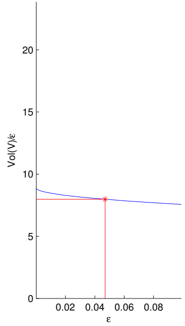
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

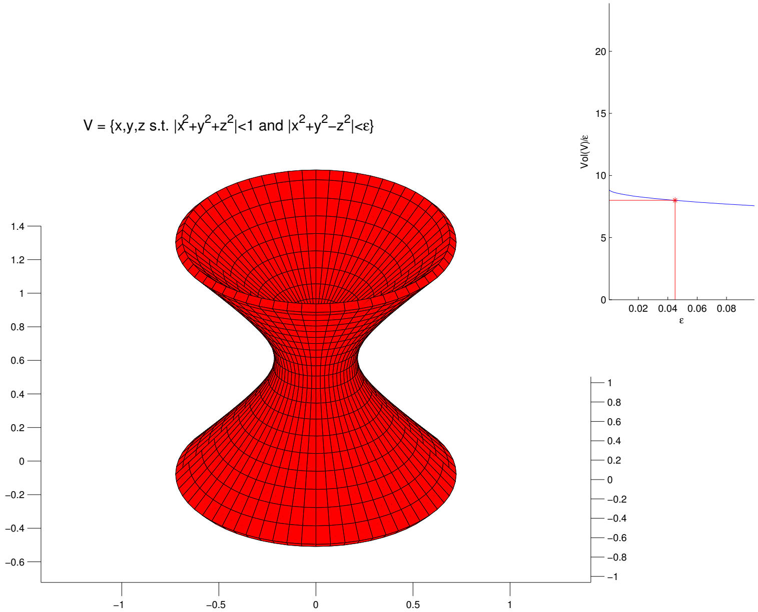




V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}






V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
0.02
0.04
0.06
0.08
0
5
10
15
20
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
0.5
1
1
0.8
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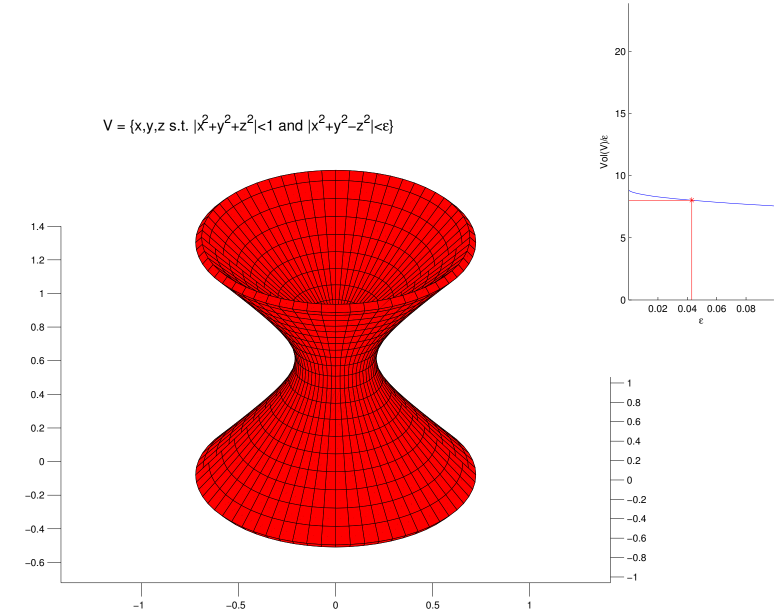
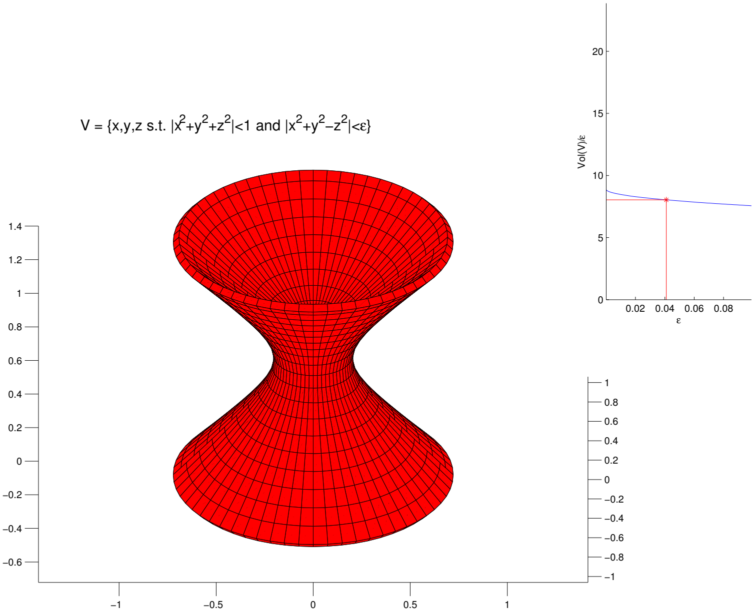
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

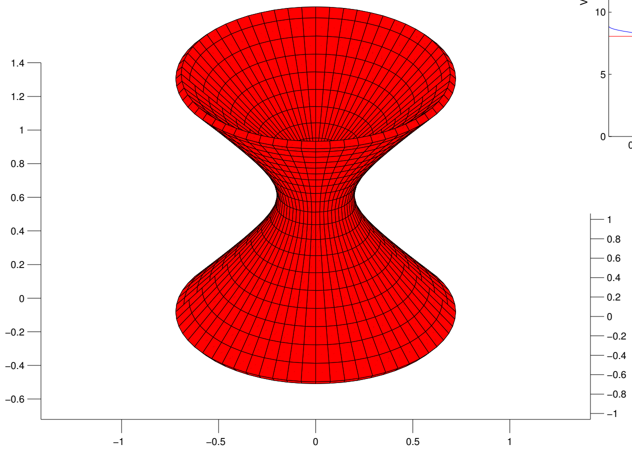


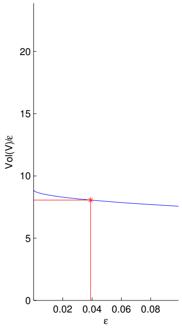
V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε



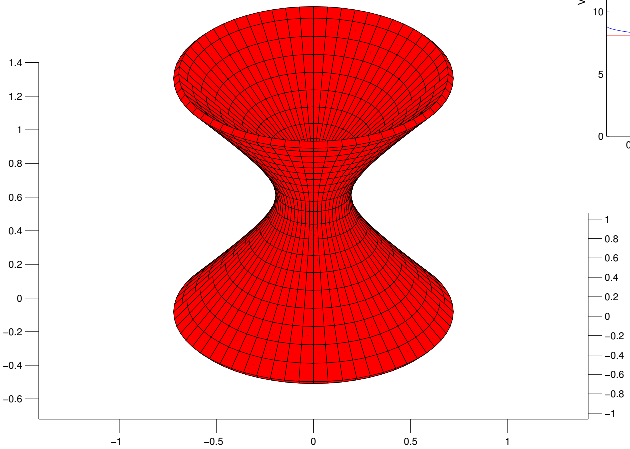
V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε

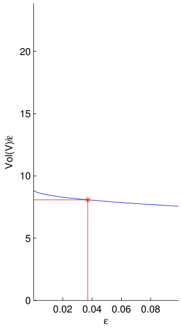
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



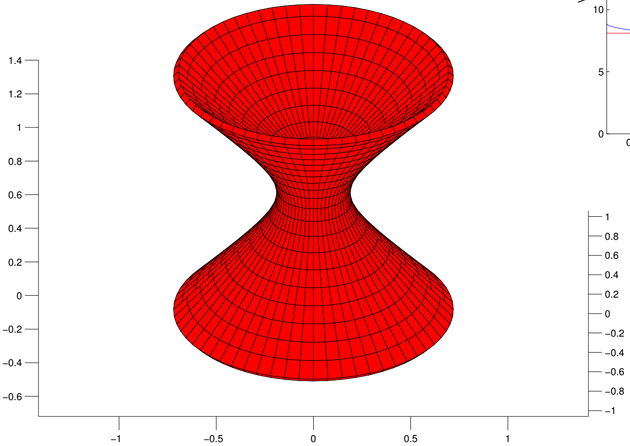


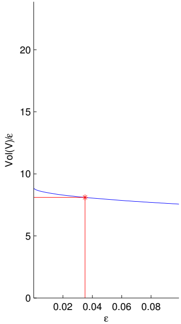
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

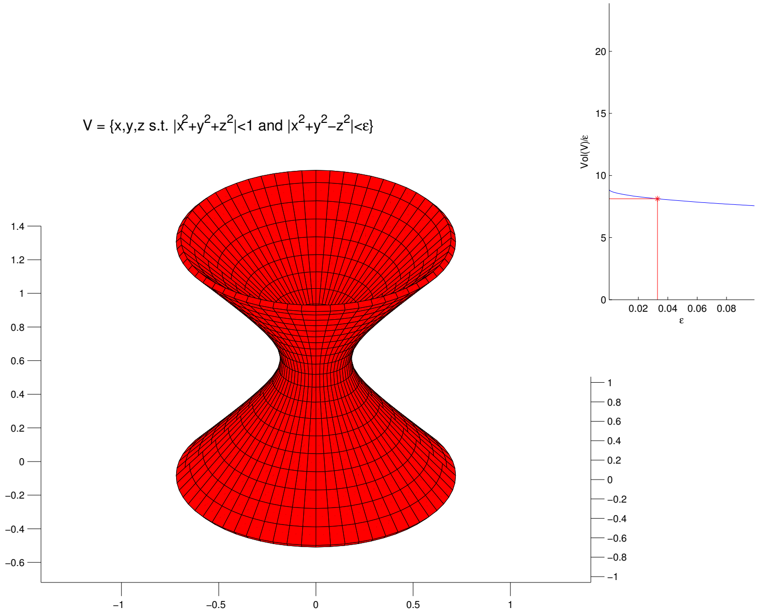




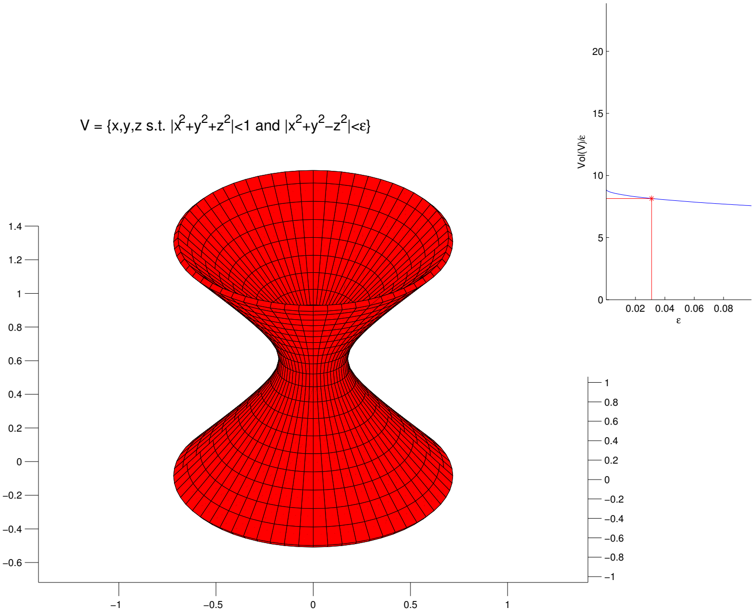
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

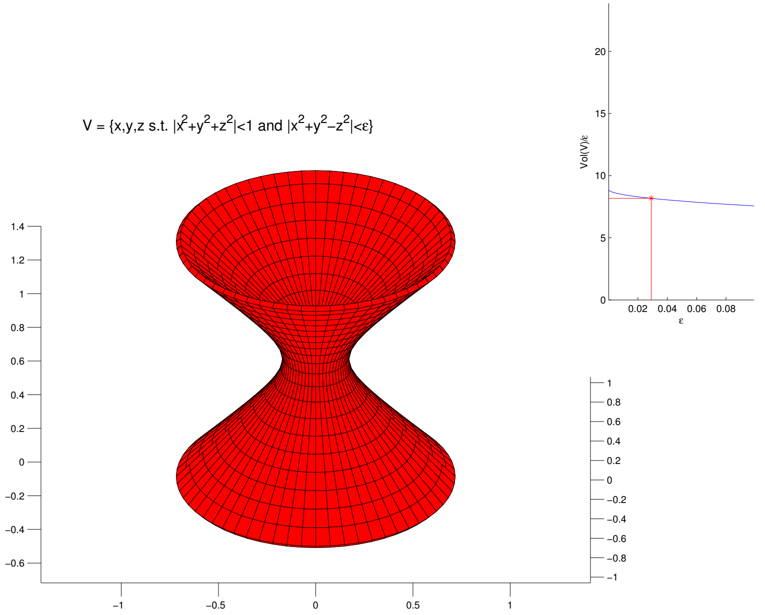






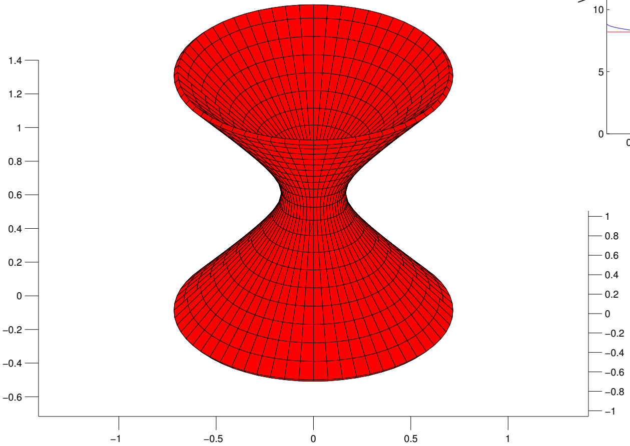
V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε

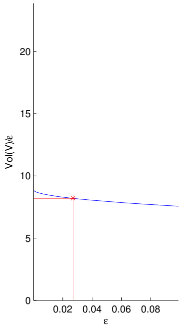

V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε
20
15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
0.5
1
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

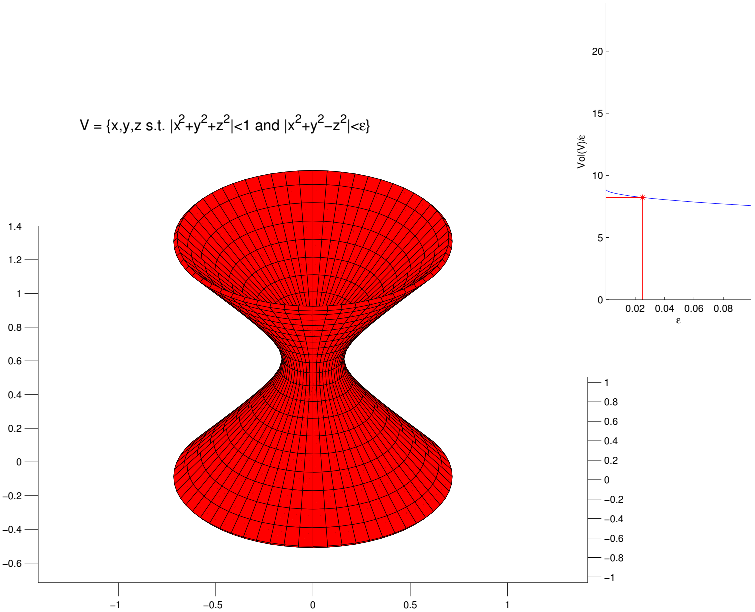


V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε

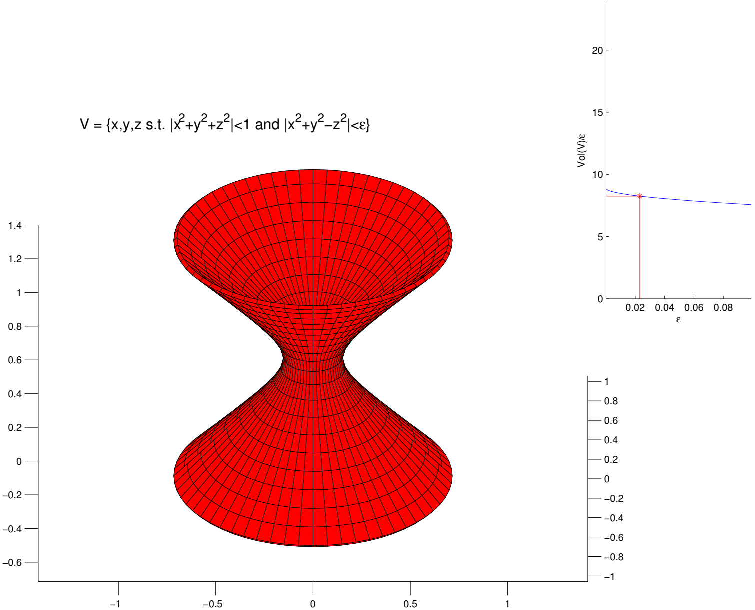
$$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$$







V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε


V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
20
15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
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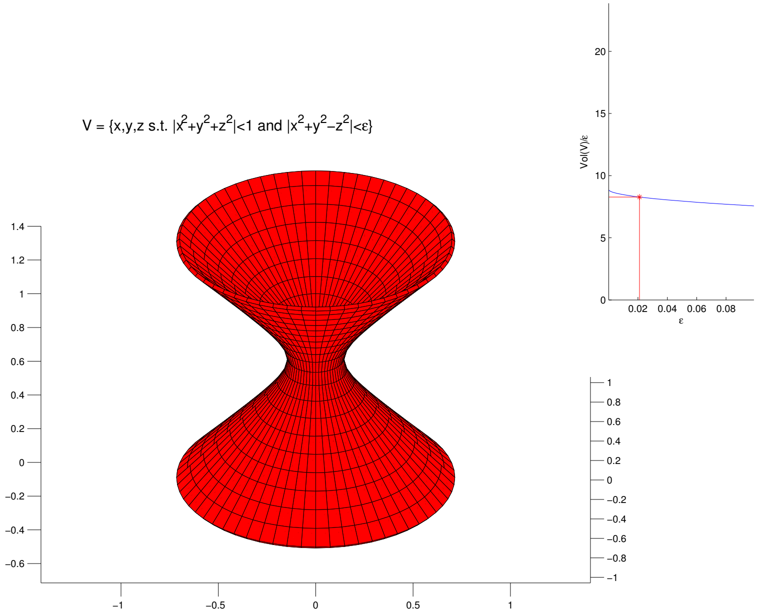
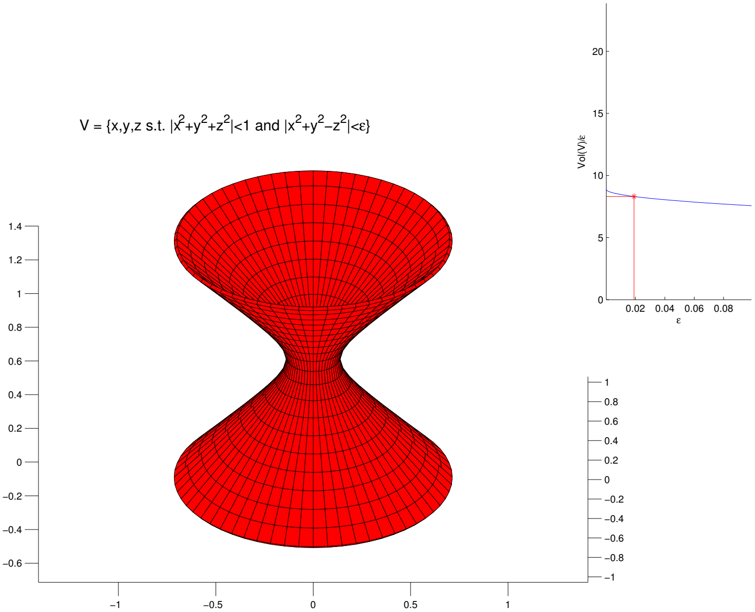
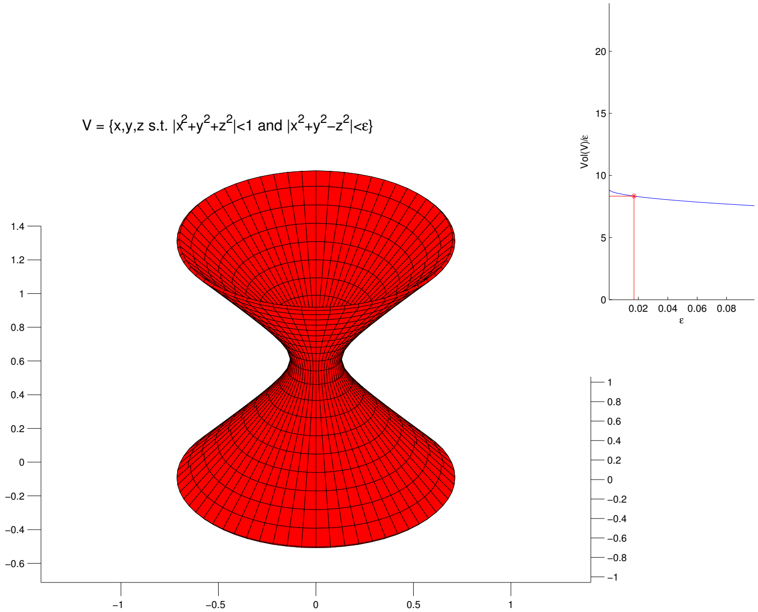
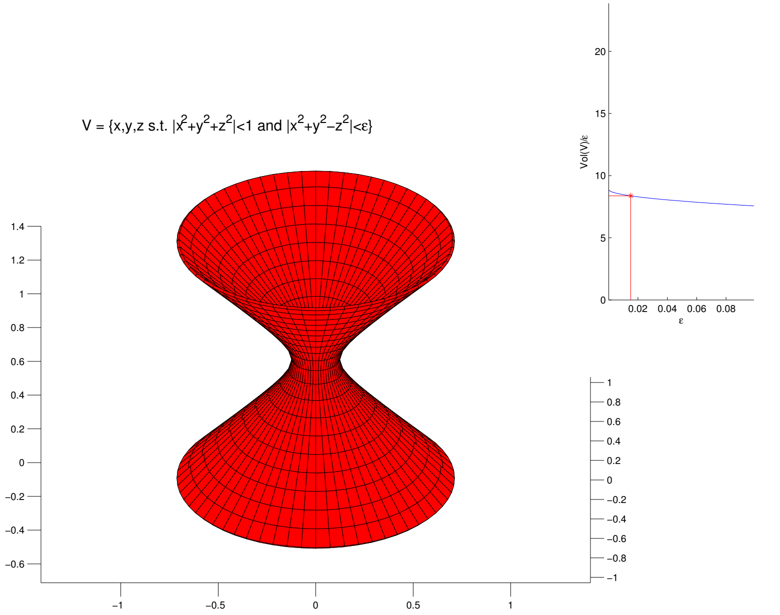


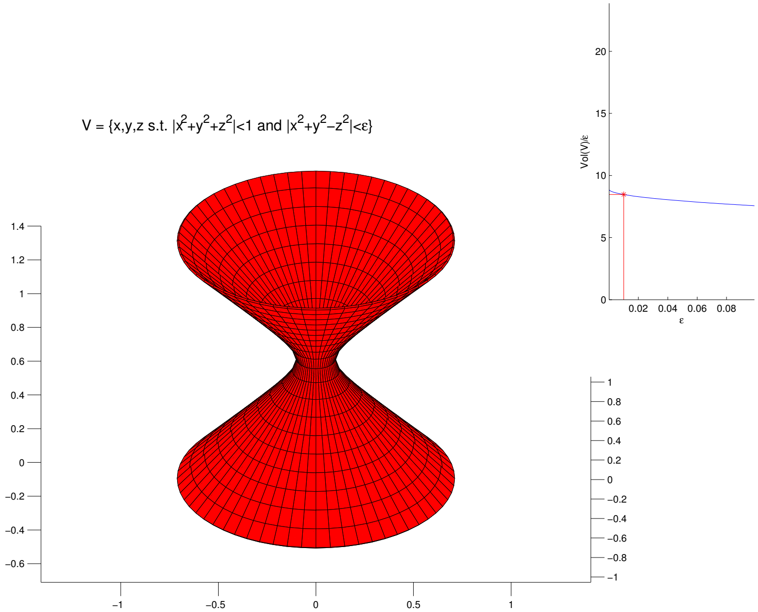
V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε


V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}
Vol(V)/ε
ε
0.02 0.04 0.06 0.08
0 5 10 15 20
1.4 1.2 1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6
−1 −0.5 0 0.5 1
1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6 −0.8 −1



V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε


V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
0
5
10
15
20
0.02
0.04
0.06
0.08


V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
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5
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0.06
0.08
1.4
1.2
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0.6
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−0.4
−0.6
−1
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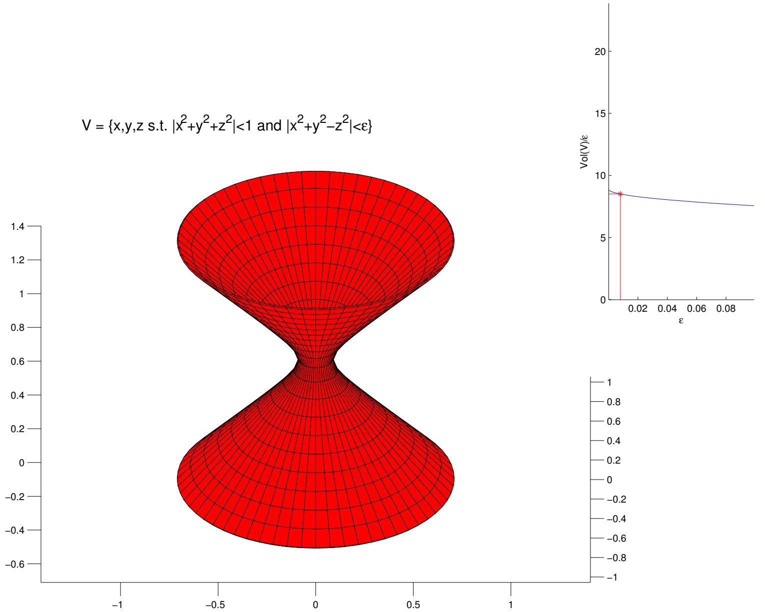
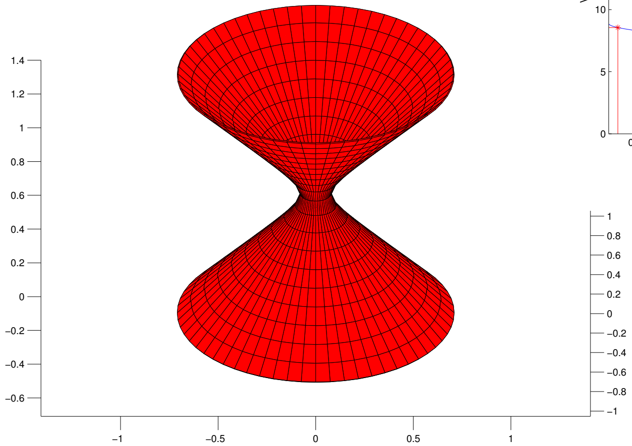
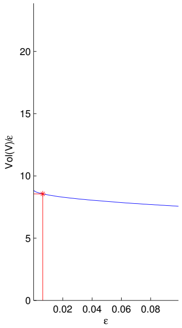
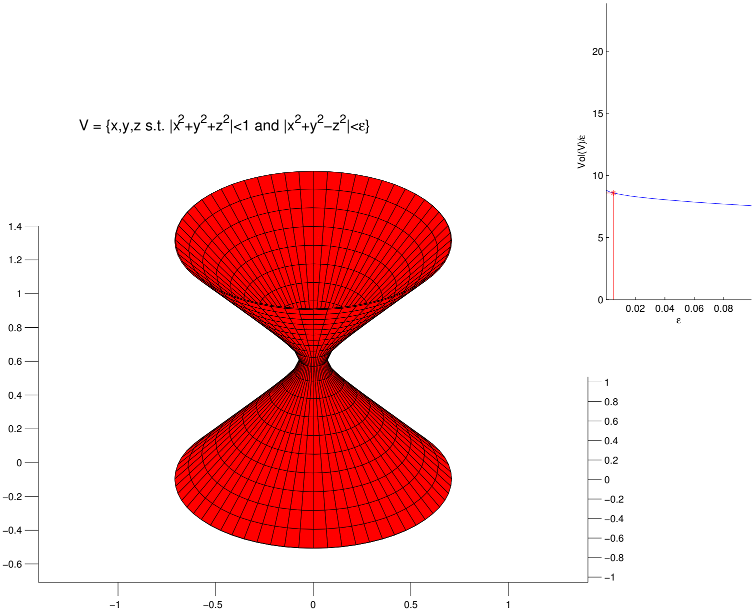
−1

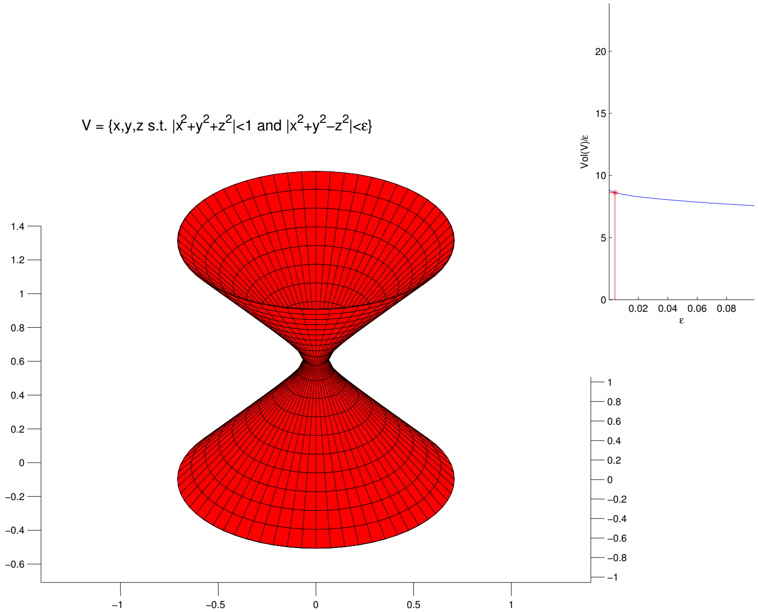

V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε

$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

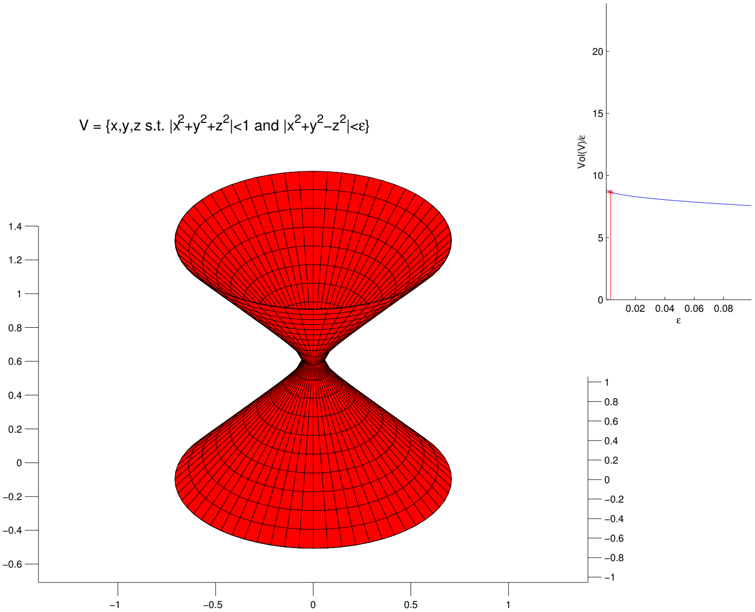




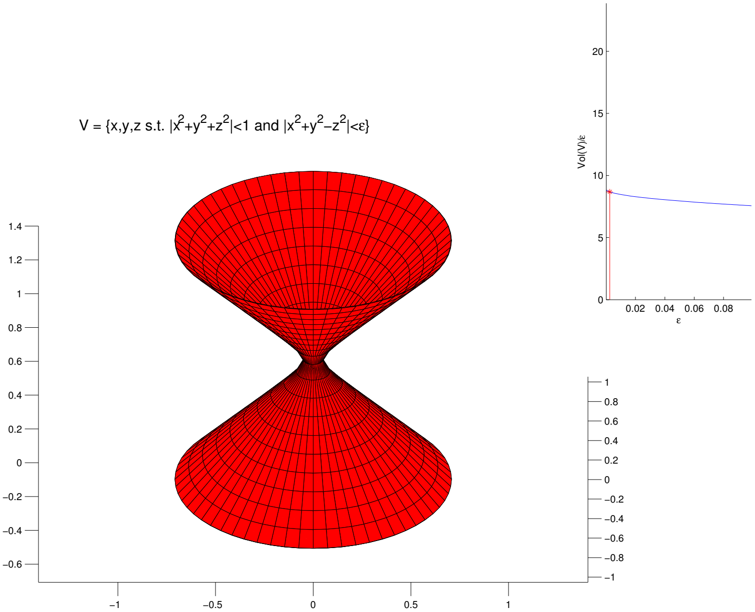

V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε



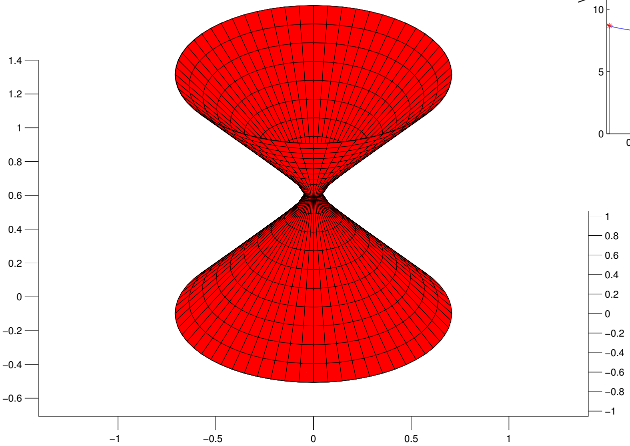
V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε

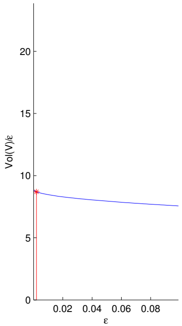

V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε

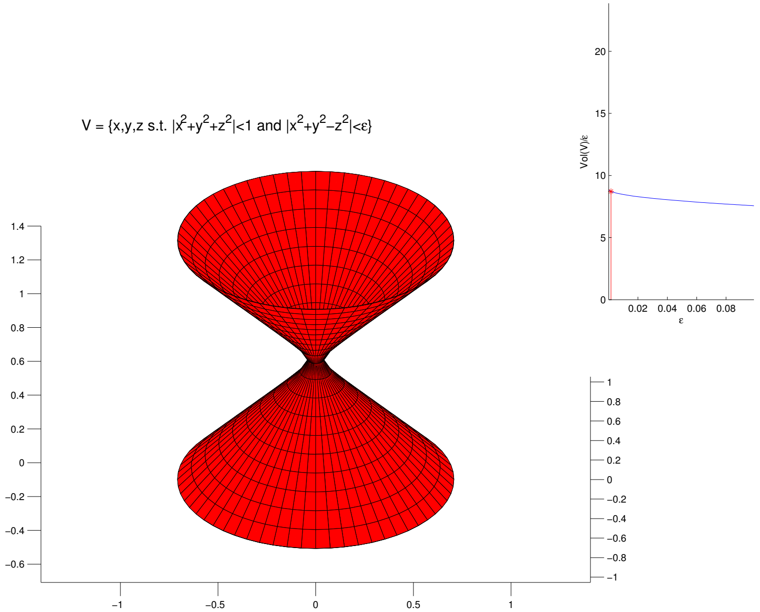
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



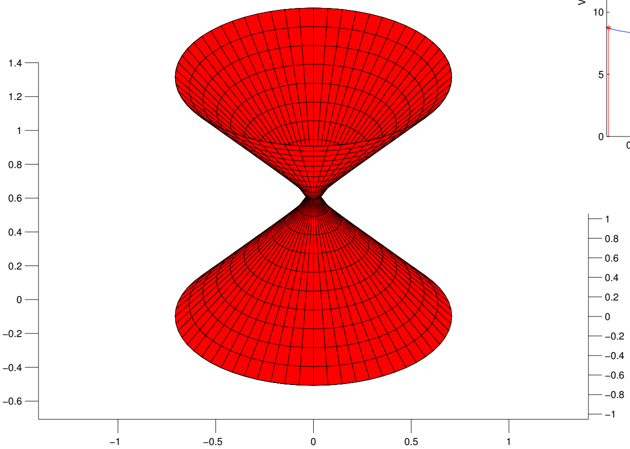
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

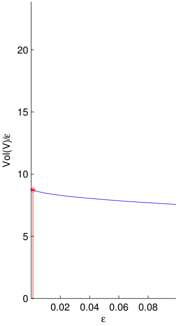


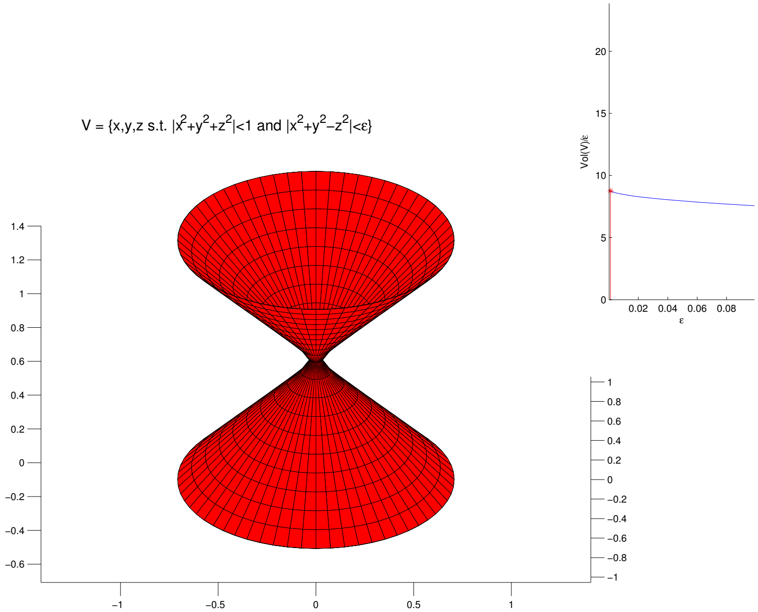


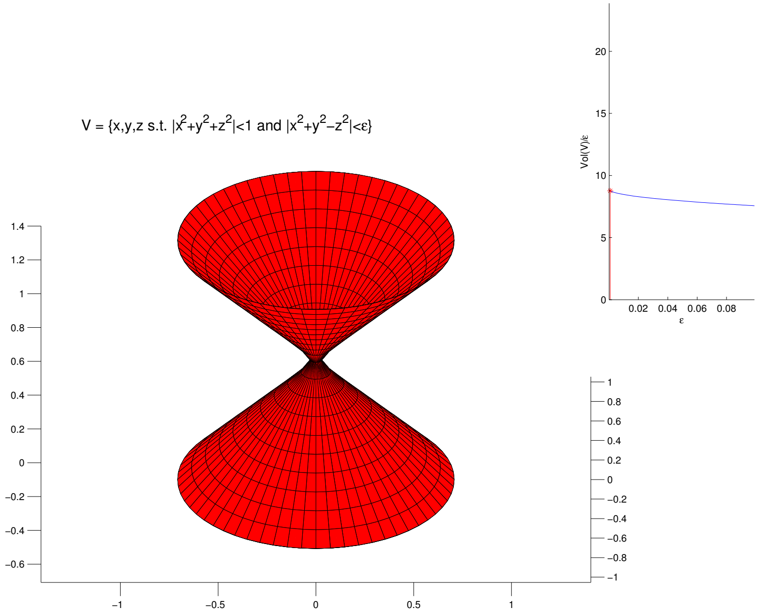

V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
0.02 0.04 0.06 0.08
0 5 10 15 20
1.4 1.2 1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6
−1 −0.5 0 0.5 1
1 0.8 0.6 0.4 0.2 0 −0.2 −0.4 −0.6 −0.8 −1

$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$






V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε


V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
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15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
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1
1
0.8
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0.2
0
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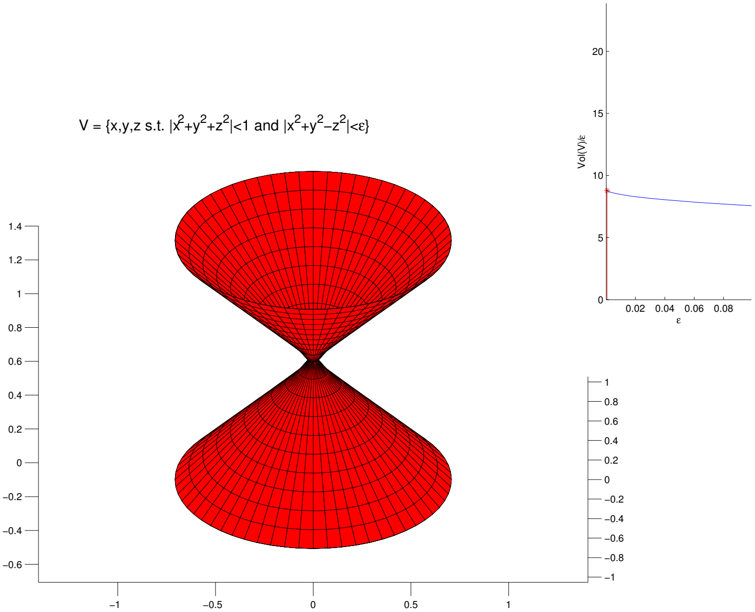
−0.4
−0.6
−0.8
−1



V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε
0
5
10
15
20
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
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0.5
1
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
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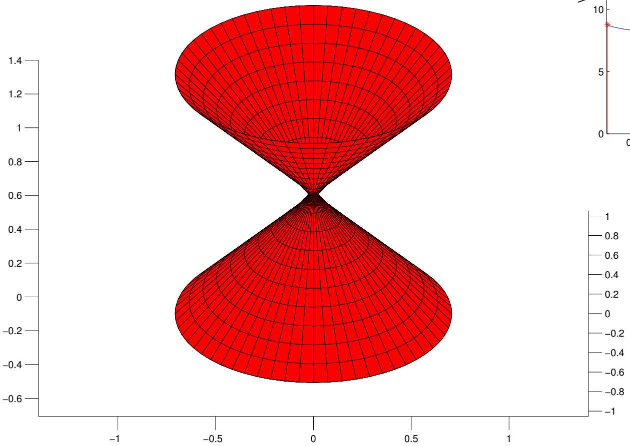
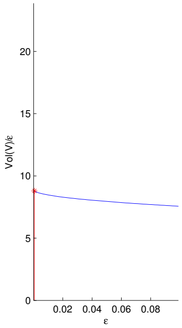
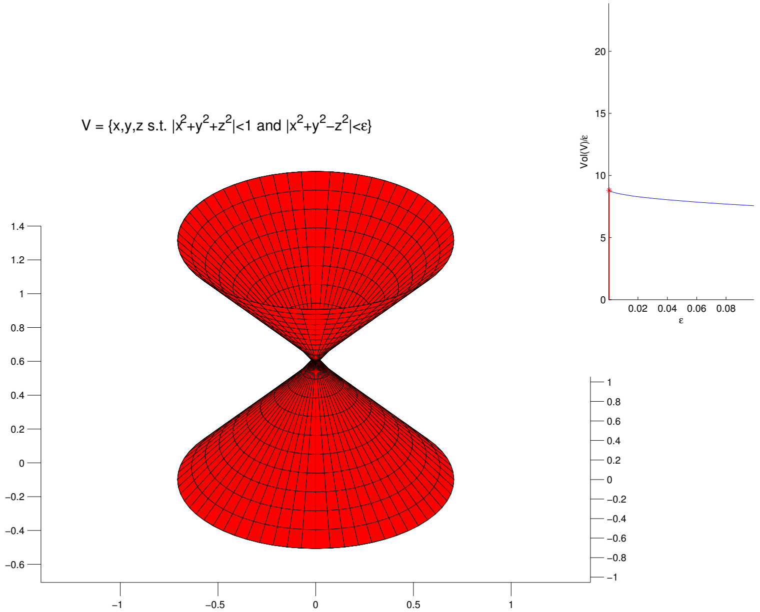
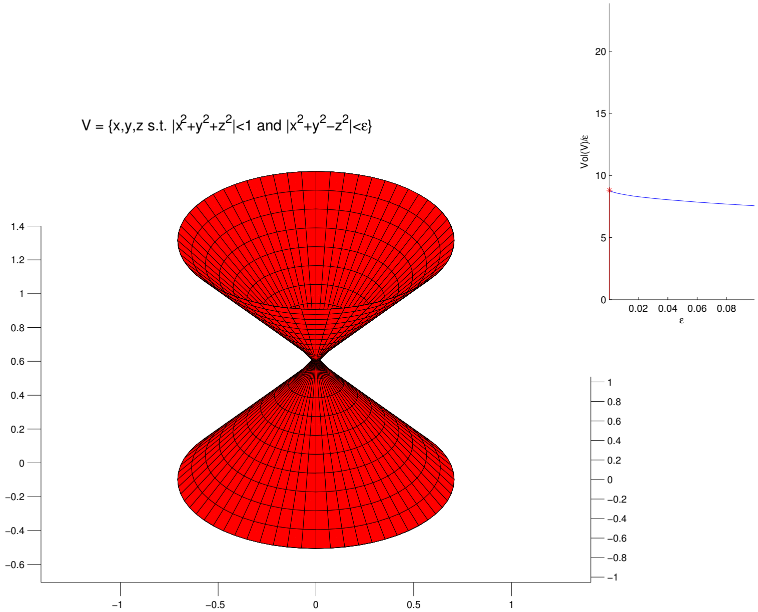
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$







V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε


V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε
0
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10
15
20
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
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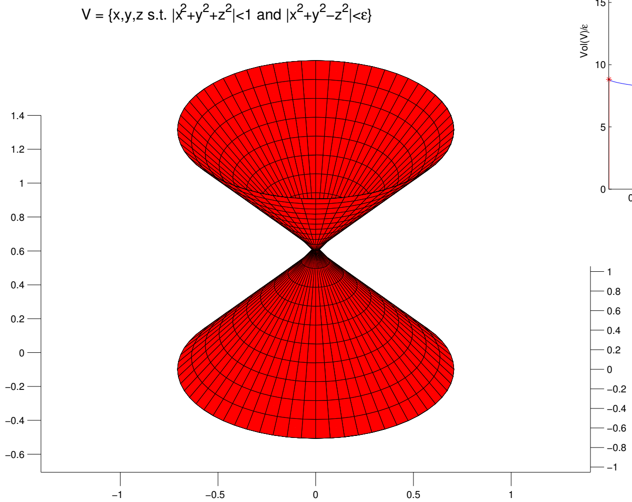
V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
1.4
1.2
1
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0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
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1
1
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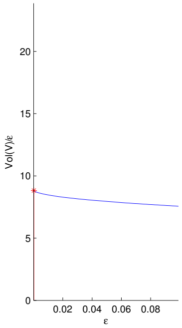
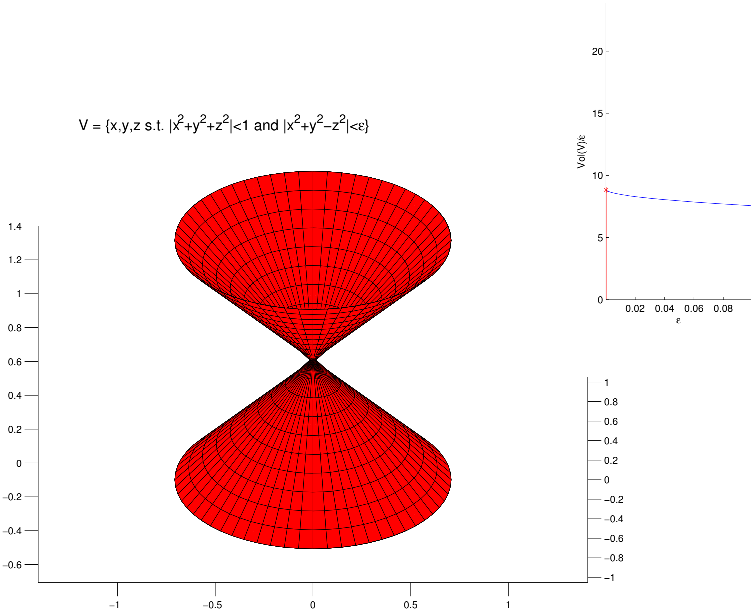
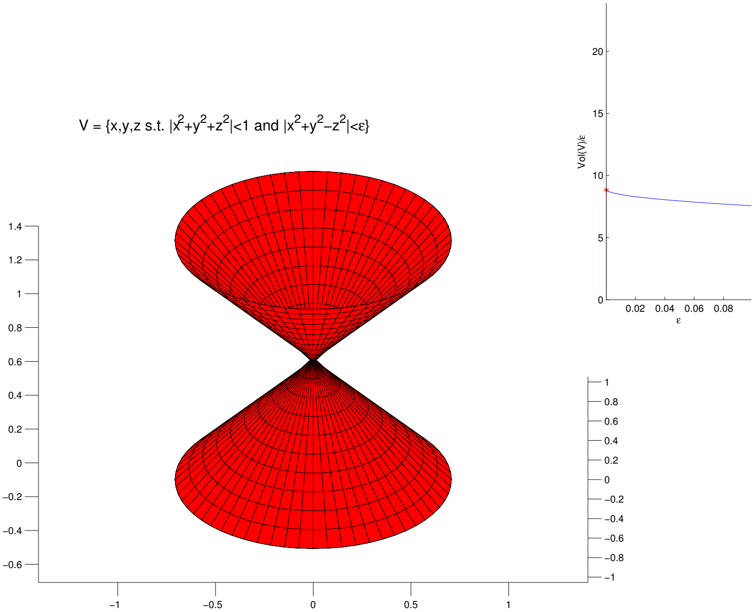
−0.4
−0.6
−0.8
−1


Vol(V)/ε
20
15
10
5
0
0.02
0.04
0.06
0.08
ε

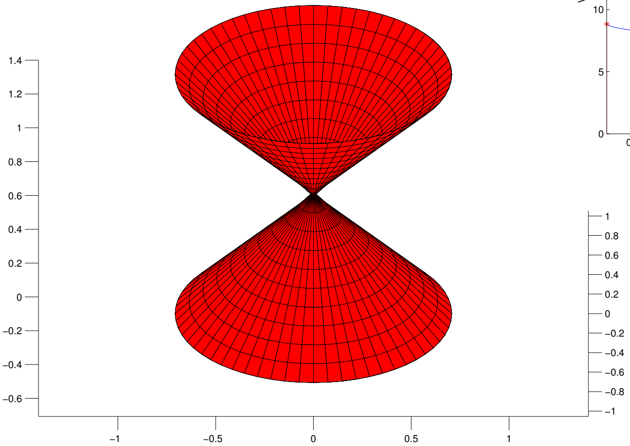


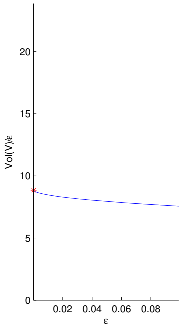
V = {x,y,z s.t. |x^2+y^2+z^2|<1 and |x^2+y^2−z^2|<ε}
Vol(V)/ε
ε



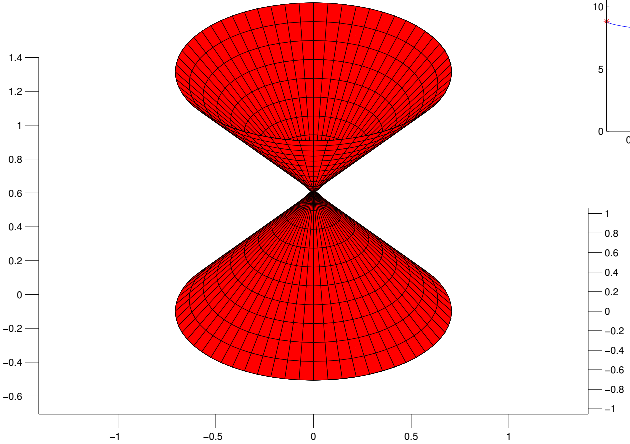
V = {x,y,z s.t. |x²+y²+z²|<1 and |x²+y²−z²|<ε}
Vol(V)/ε
ε

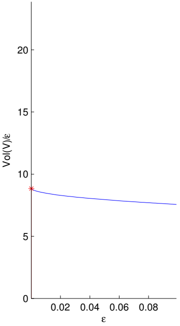
V = {x,y,z s.t. $|x^2+y^2+z^2|<1$ and $|x^2+y^2-z^2|<\varepsilon$}



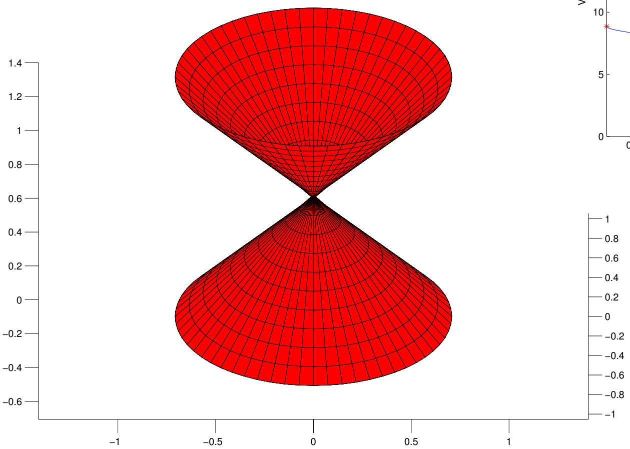


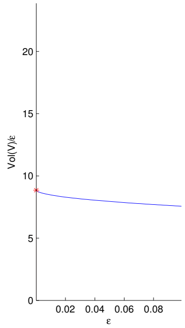
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



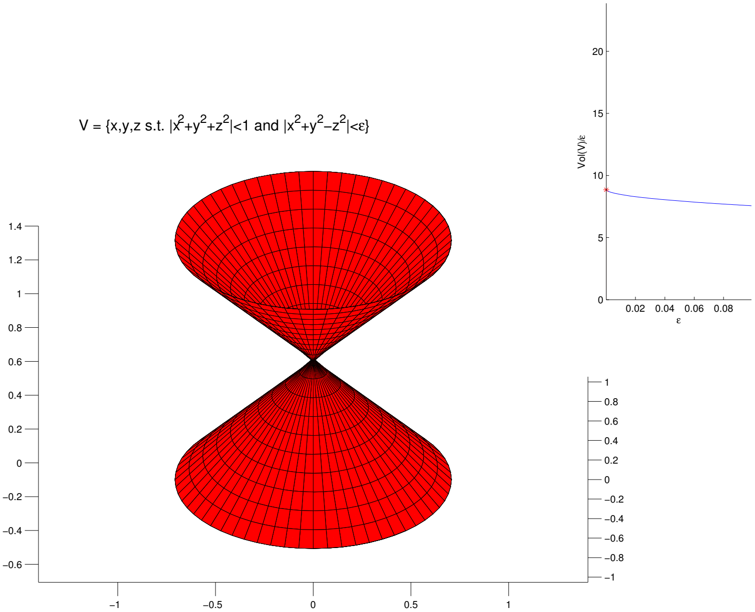


$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$

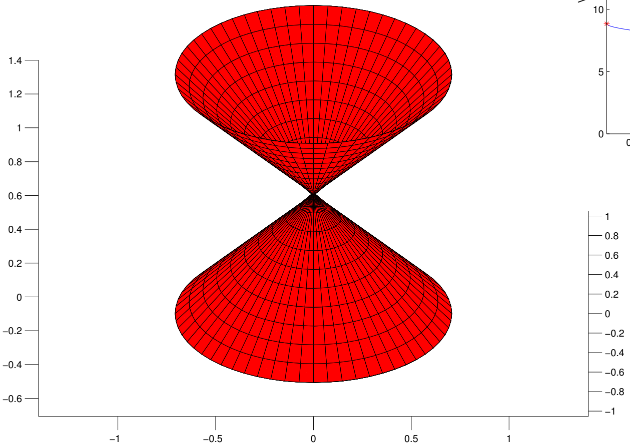


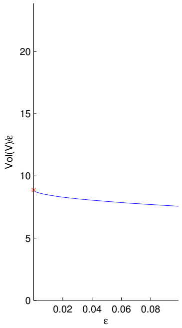


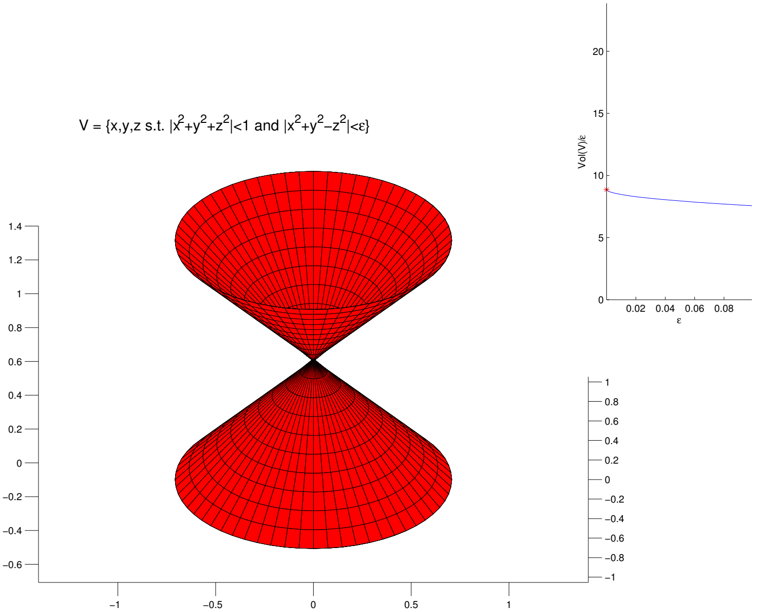
$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$



$V = \{x,y,z \text{ s.t. } |x^2+y^2+z^2|<1 \text{ and } |x^2+y^2-z^2|<\varepsilon\}$







V = {x,y,z s.t. |x2+y2+z2|<1 and |x2+y2−z2|<ε}
Vol(V)/ε
ε
20
15
10
5
0
0.02
0.04
0.06
0.08
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1
−0.5
0
0.5
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

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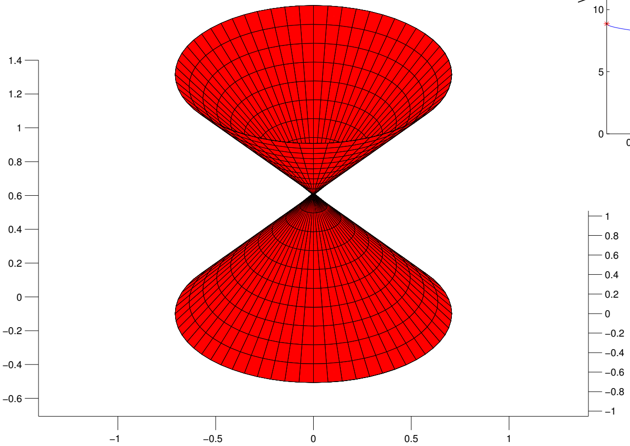

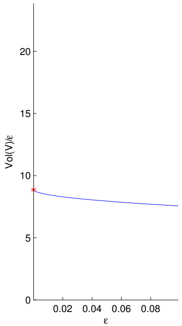

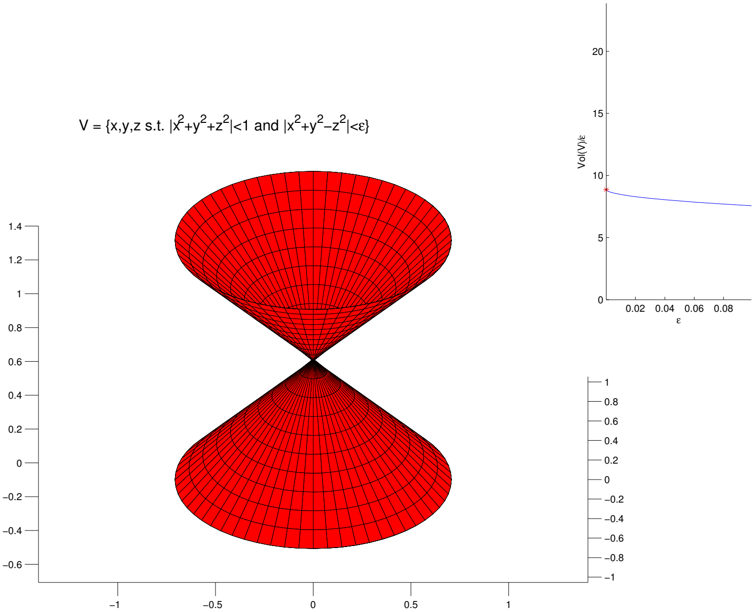
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1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
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−1
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0.5
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Vol(V)/ε
20
15
10
5
0
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0.04
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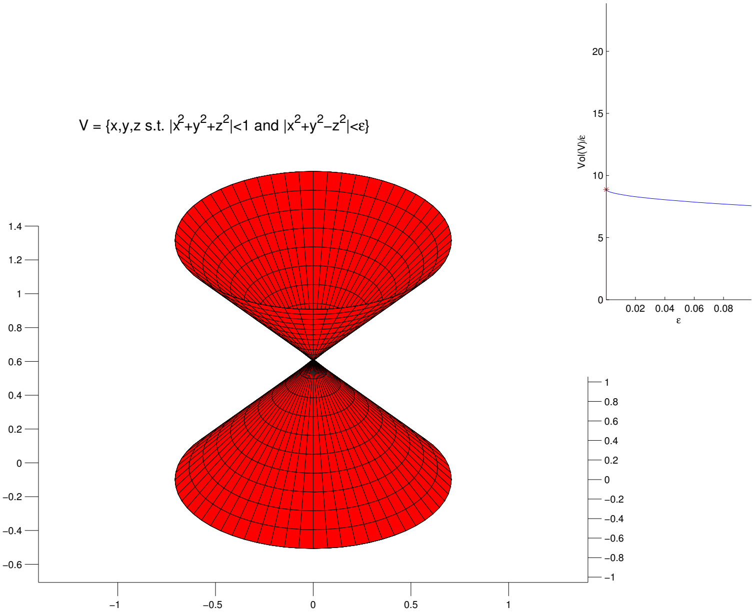
ε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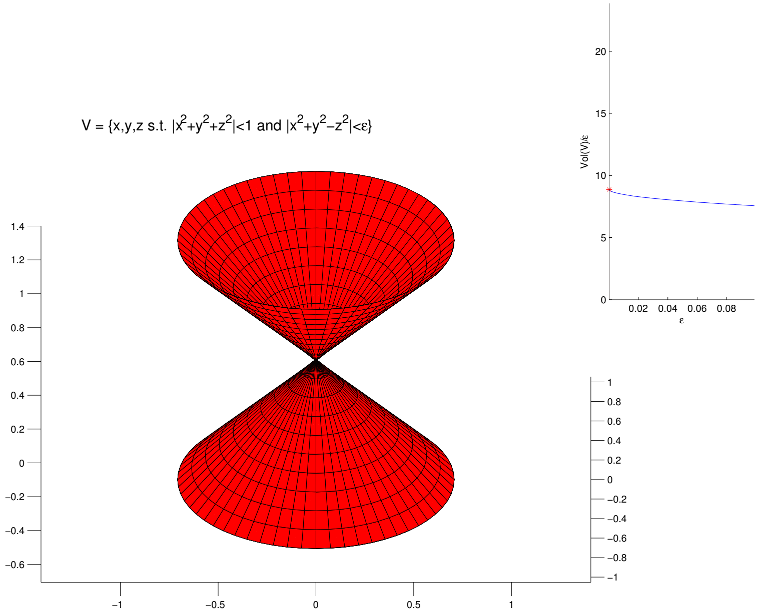

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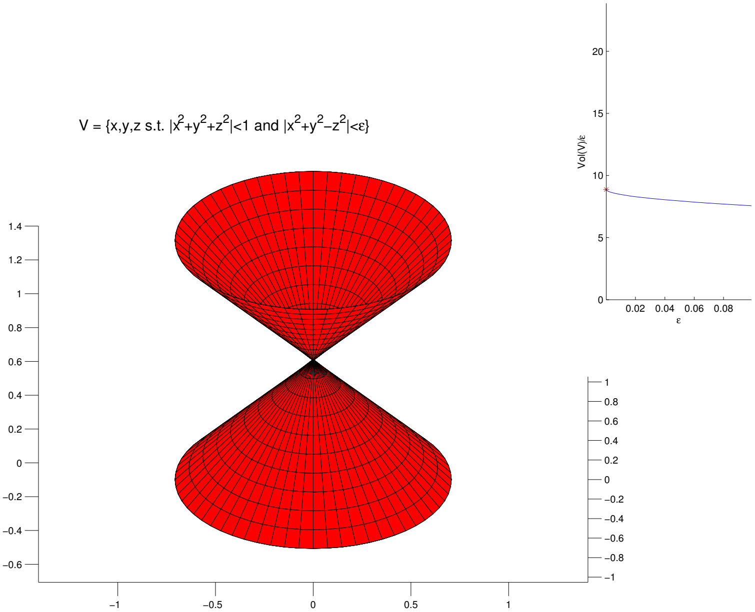
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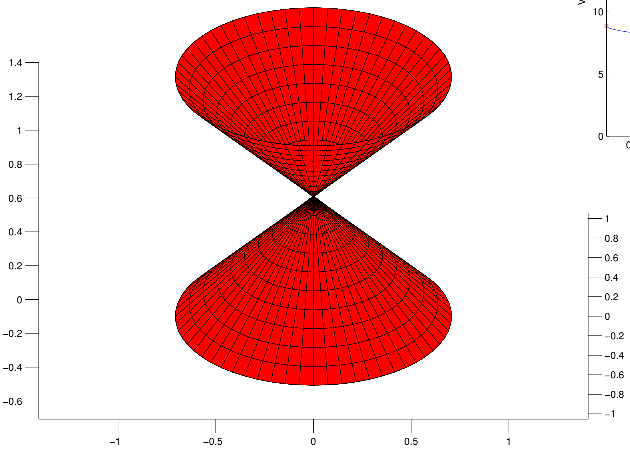

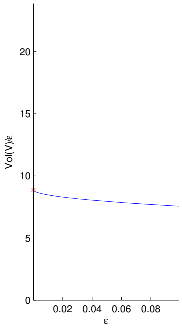

Rationality of the singularities of deformation varieties

Theorem (A.-Avni)

Let $n \geq d''$. Then Φ is FRS. In particular, the singularities of the deformation variety $\mathrm{Def}_{G,\Sigma_n}$ are rational and complete intersection.

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- Similar result for the nilpotent cone is due to Kostant and Hesselink.
- In our case we do not have an explicit resolution of singularities, unlike other cases of proof of rationality of singularity of varieties from algebraic group theory.

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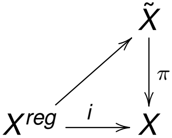

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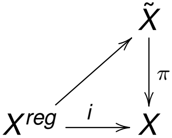

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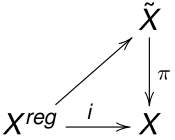

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Application: Hinich Theorem $\Longrightarrow$ Deligne Ranga-Rao Theorem

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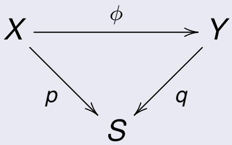

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Example (Elimination – Telescope)

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Corollary (Degeneration)

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Example (Linearization – microscope)

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Assume that $\phi^*(m_y) \subset m_x^r$. Suppose that $\sigma_r\phi : T_xX \to T_yY$ is FRS at 0. Then ϕ is FRS at x.

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Definition

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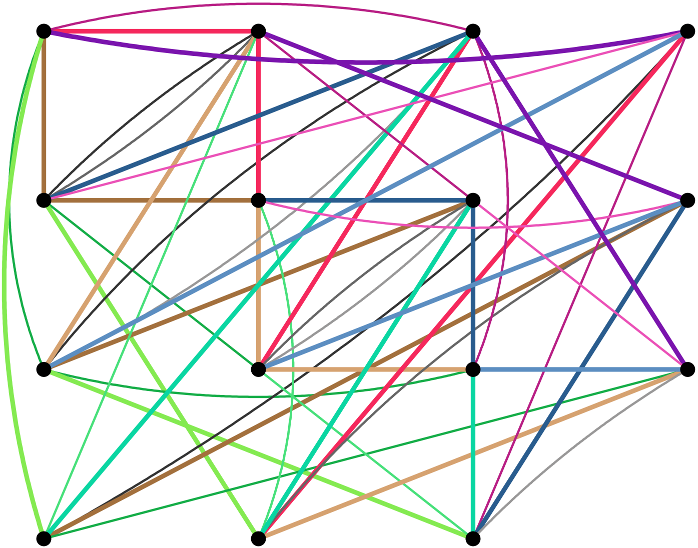

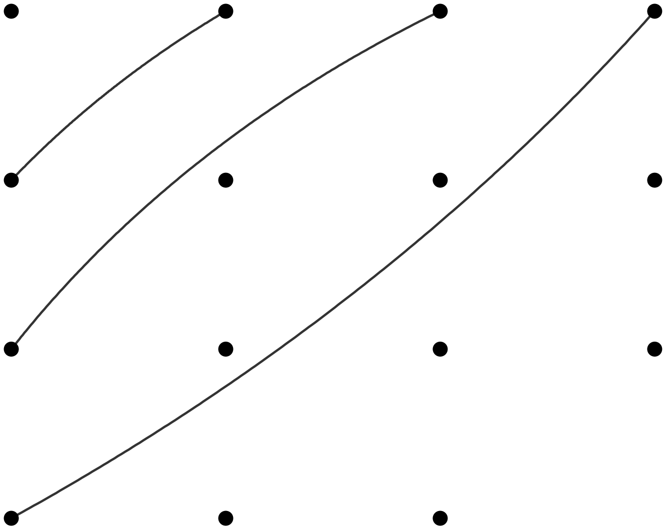

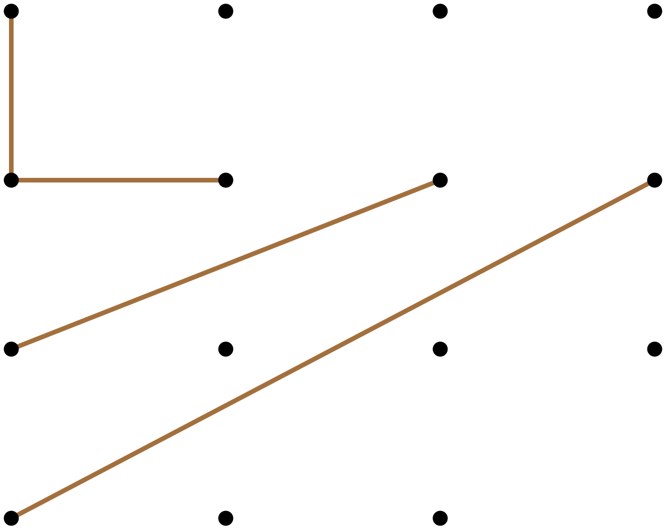

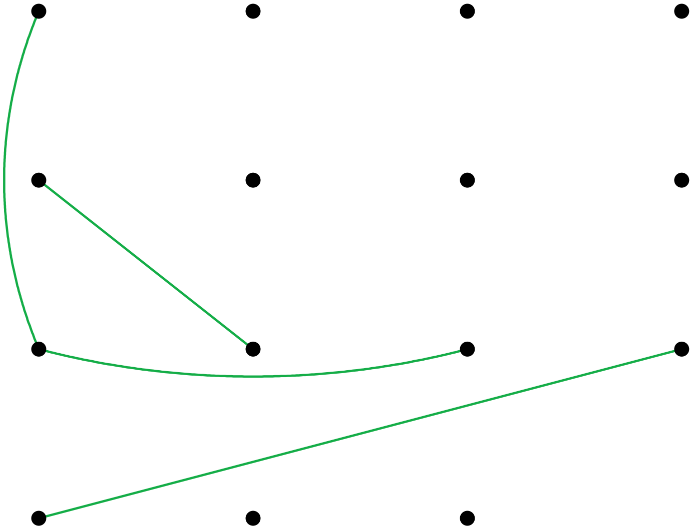

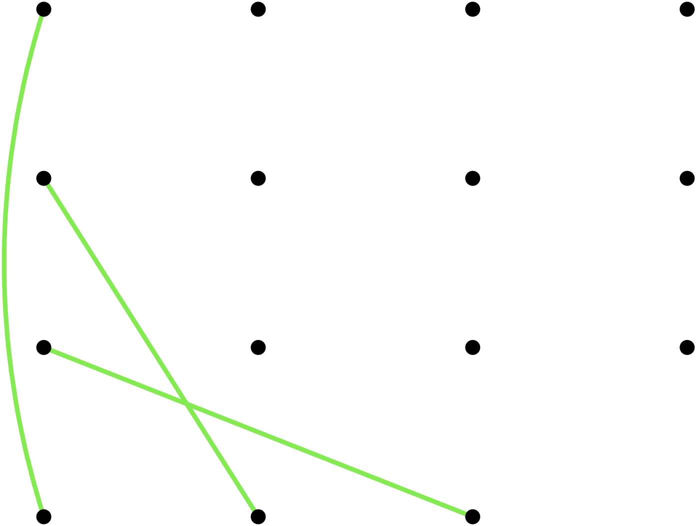

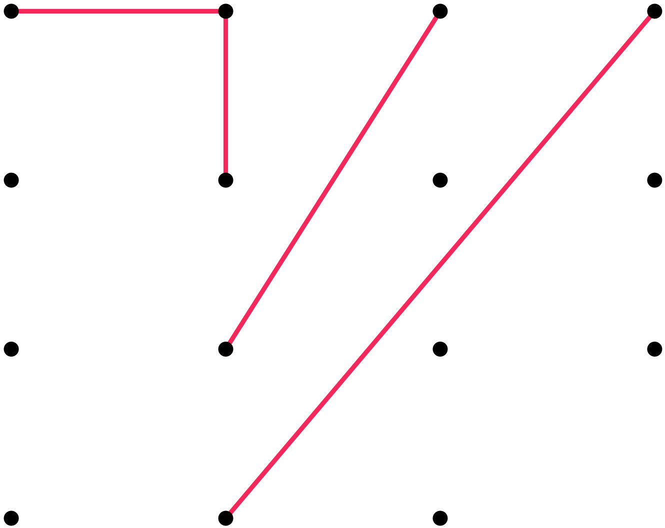

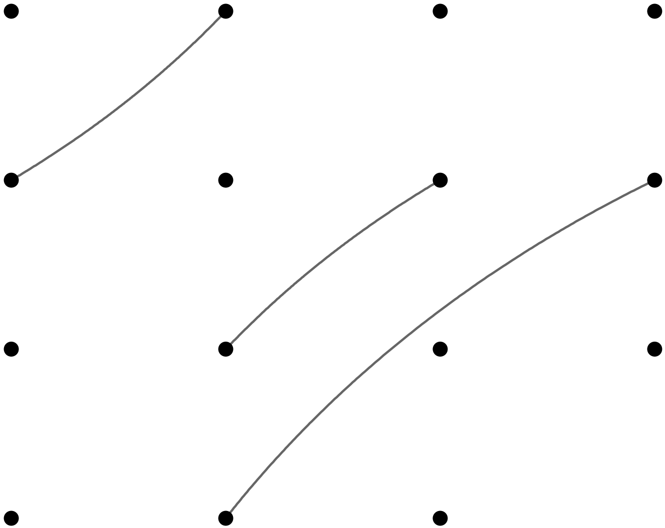

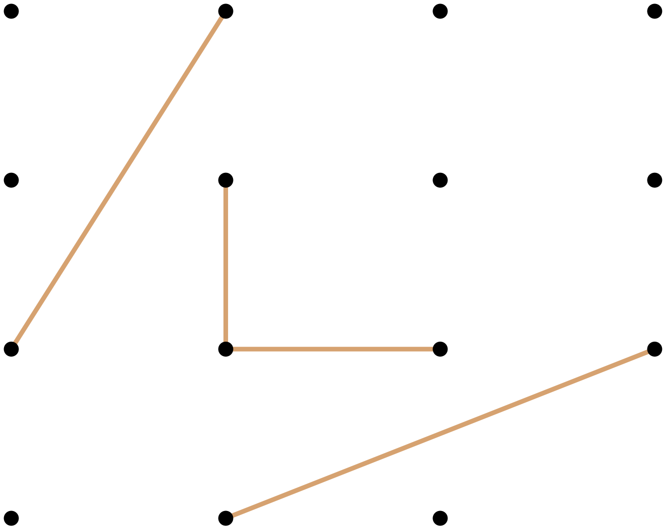

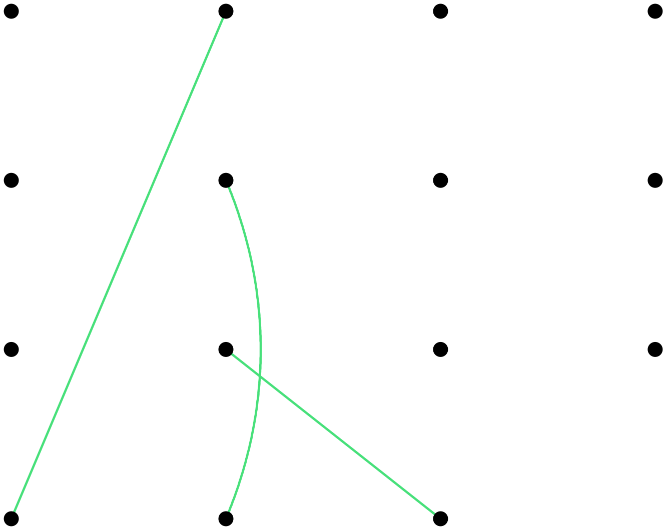

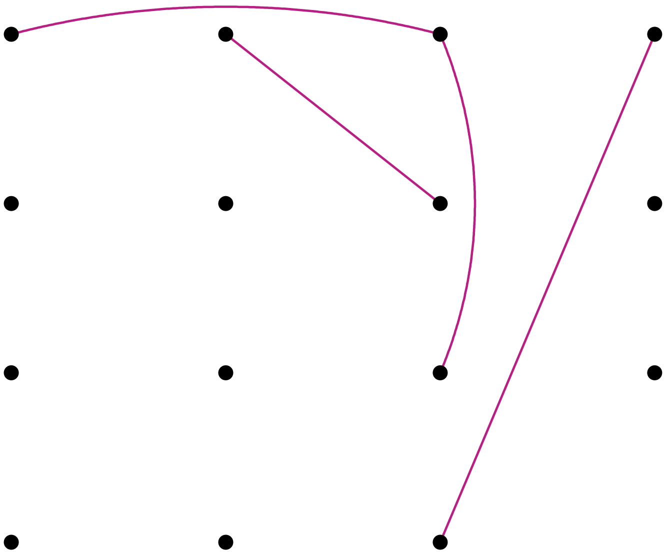

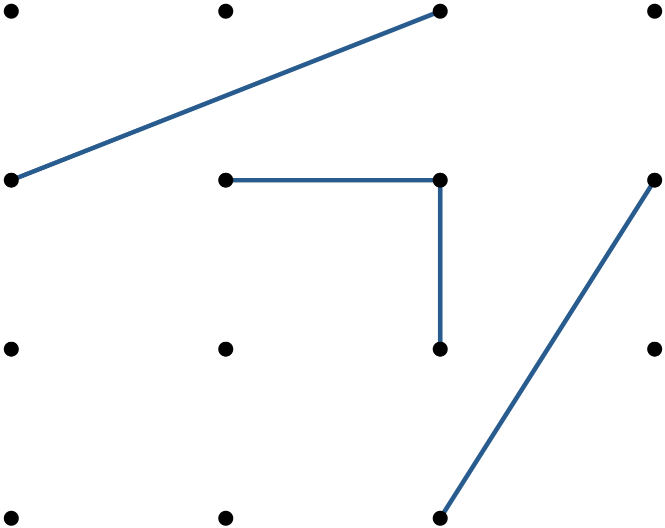

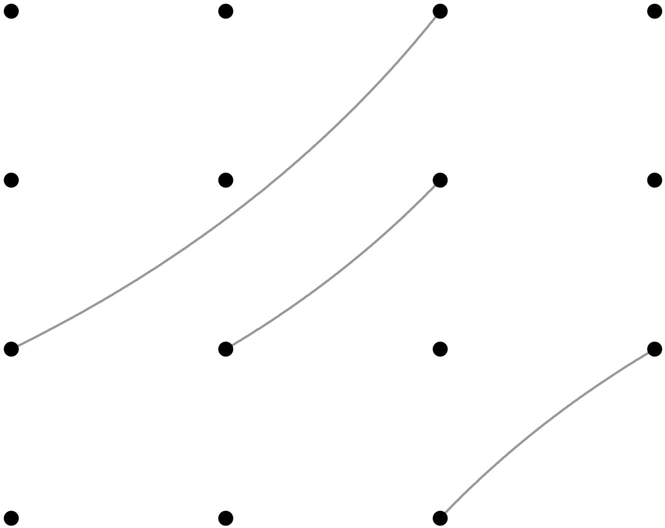

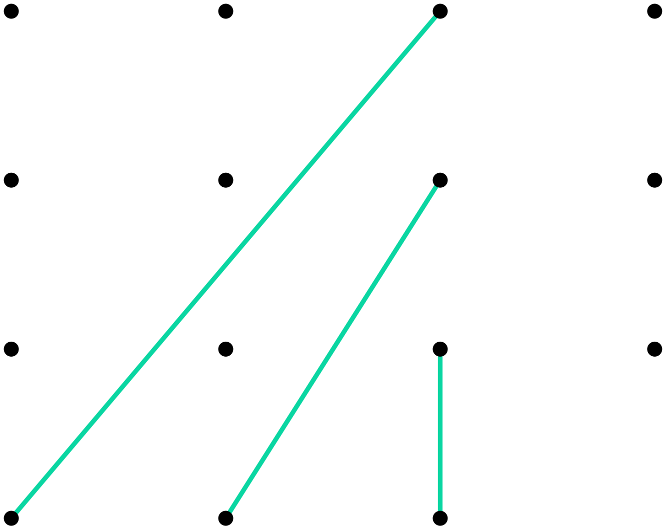

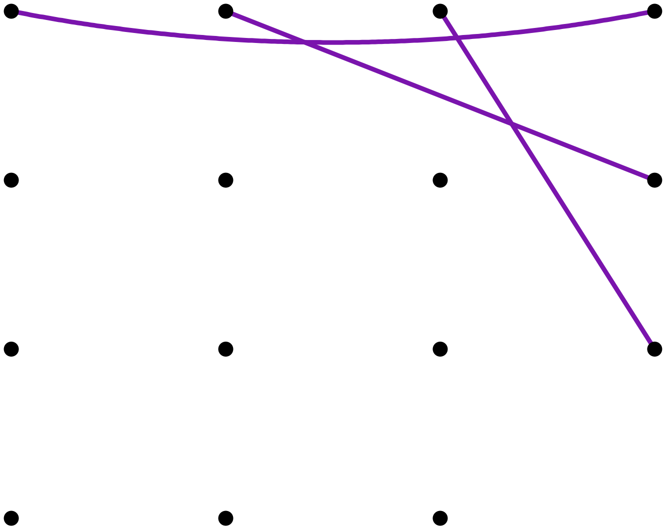

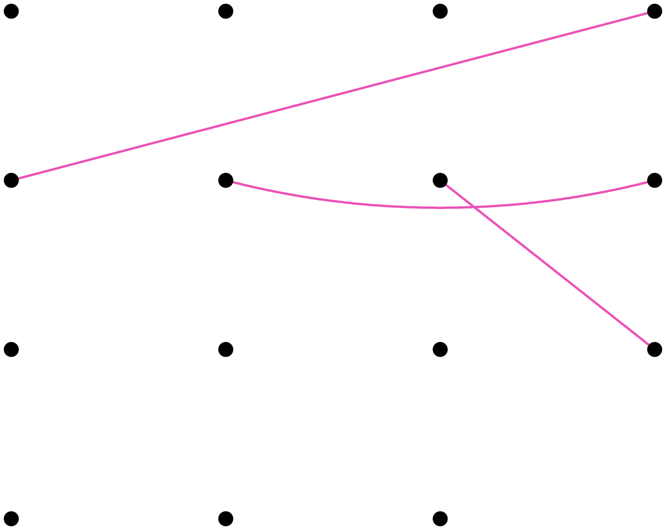

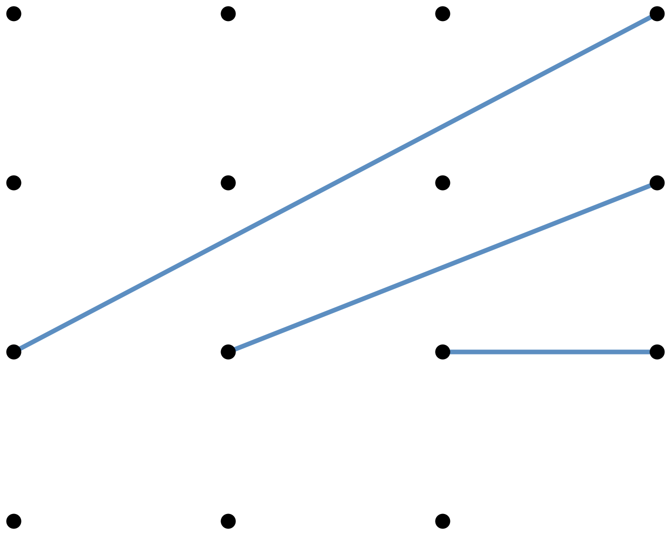

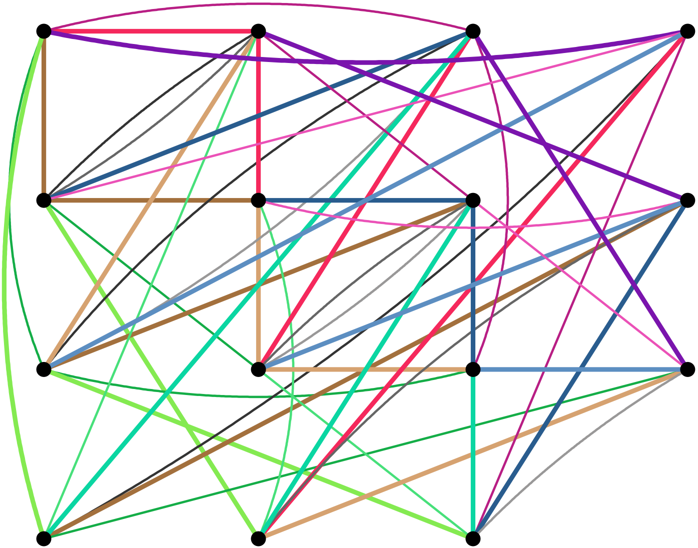

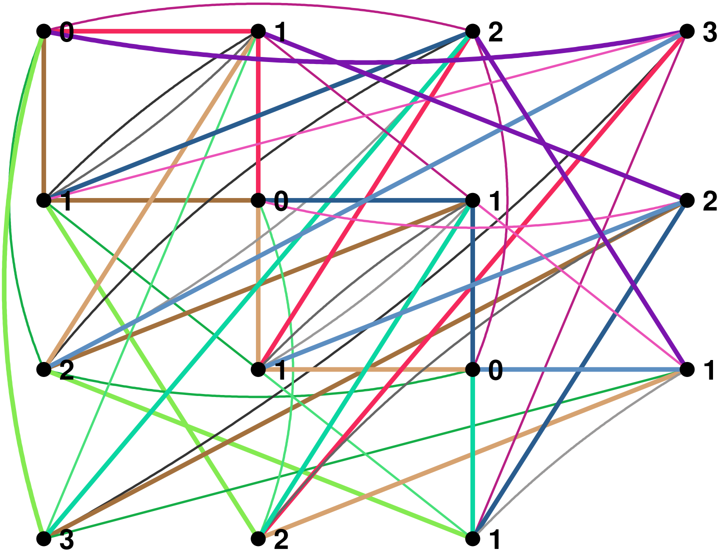
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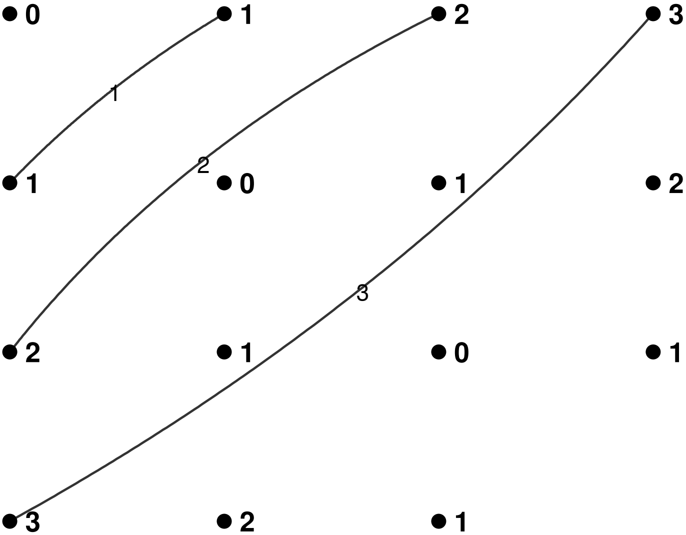
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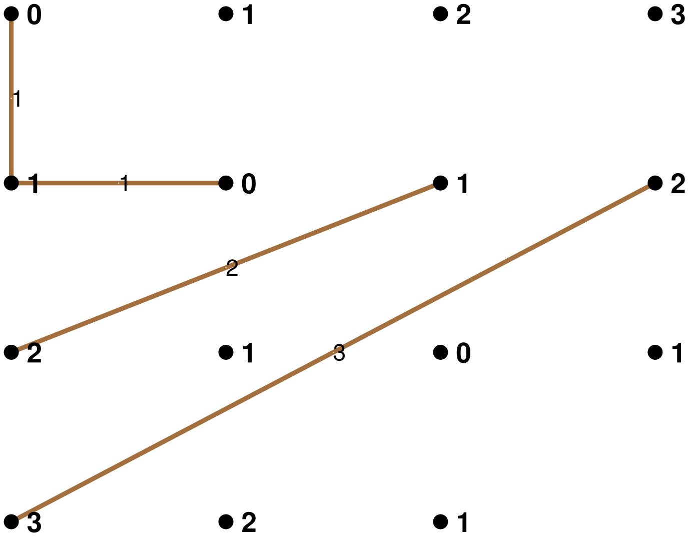

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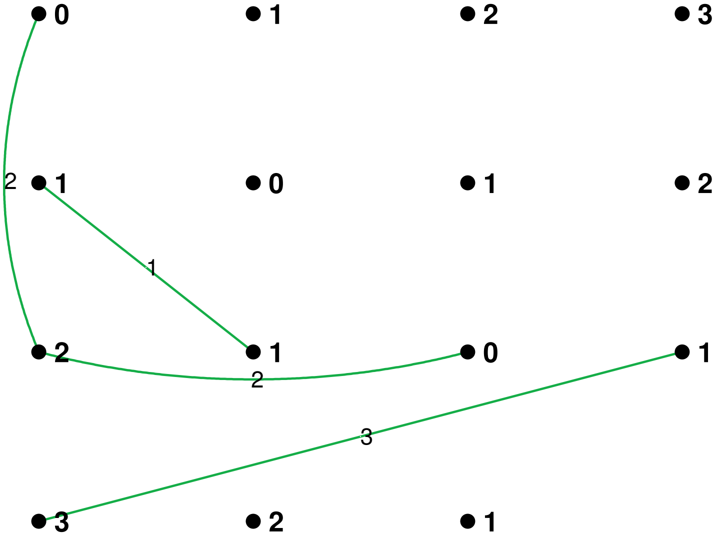
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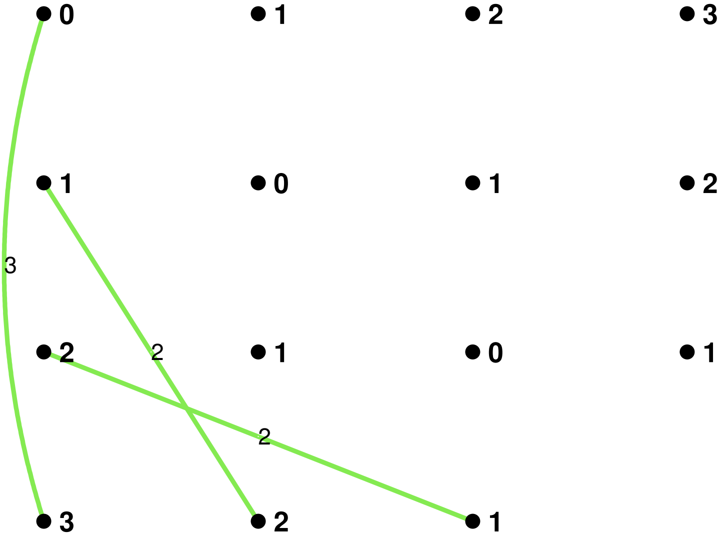
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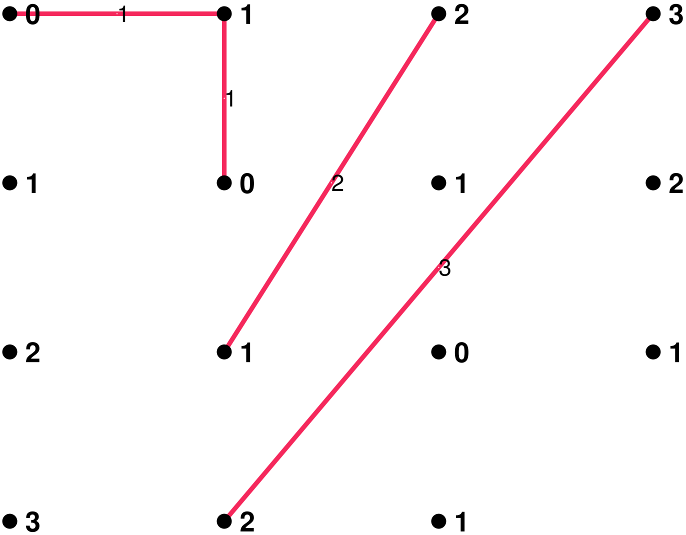
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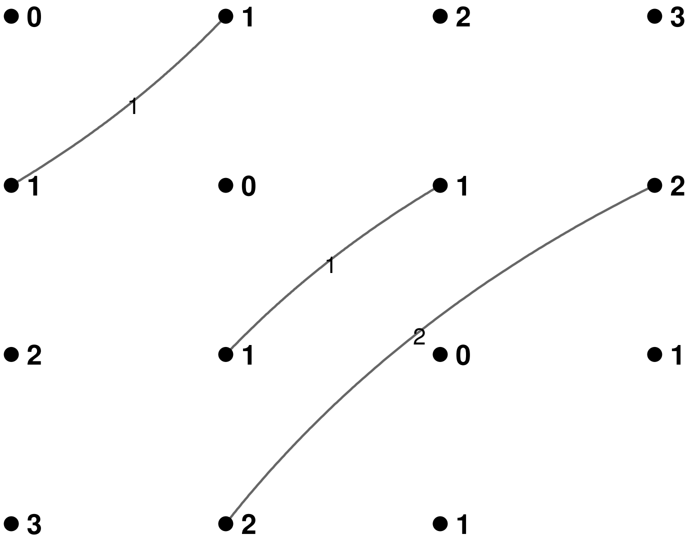
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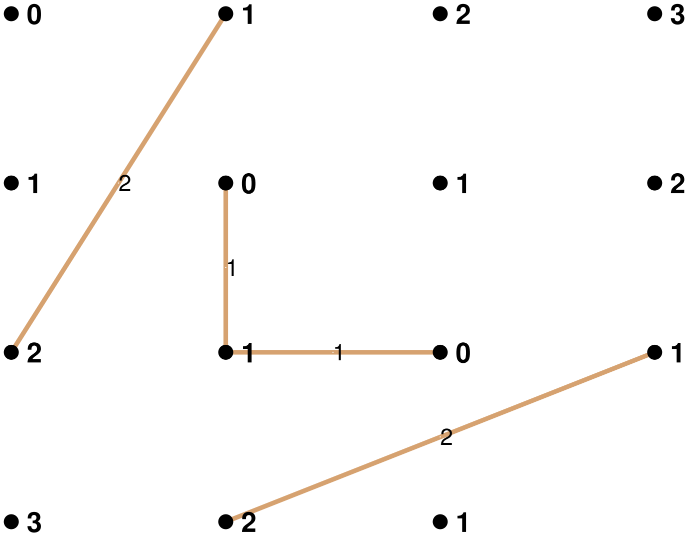
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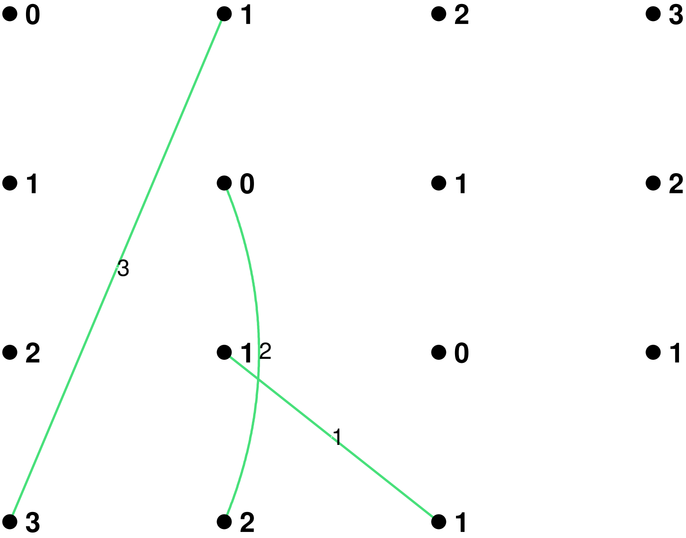

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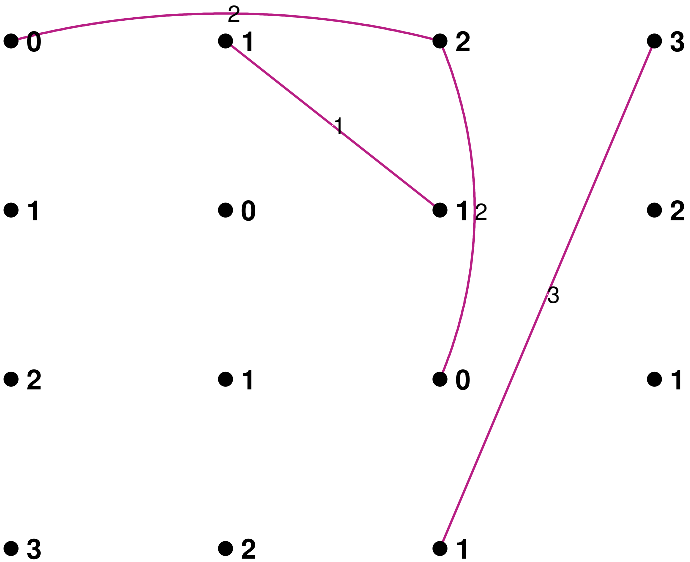
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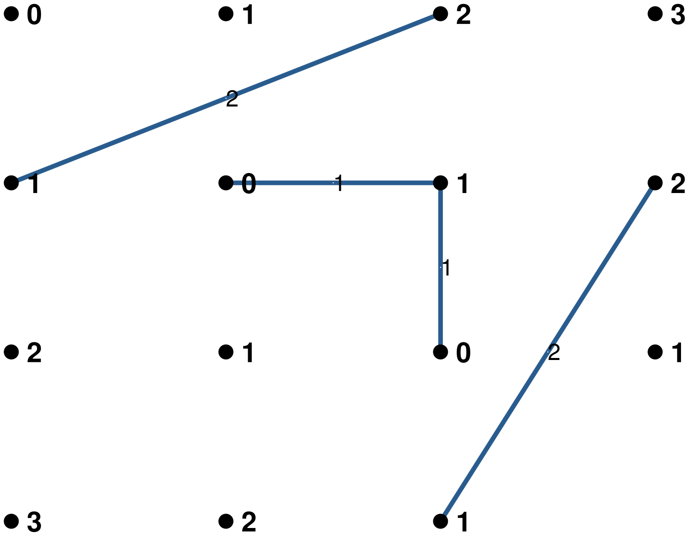
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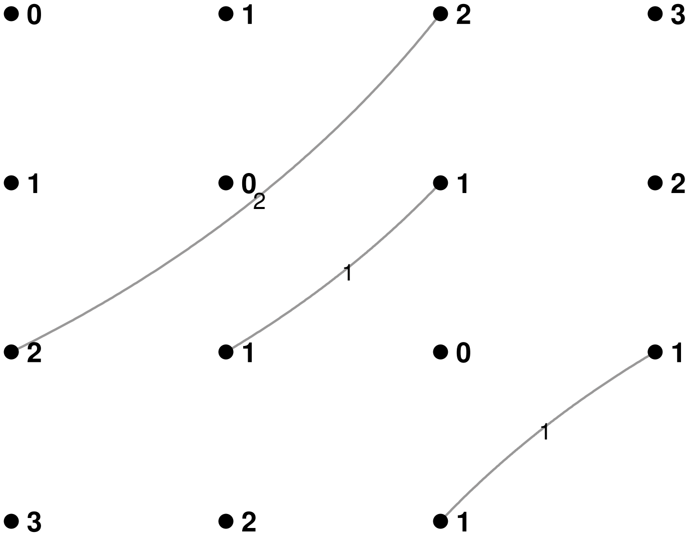
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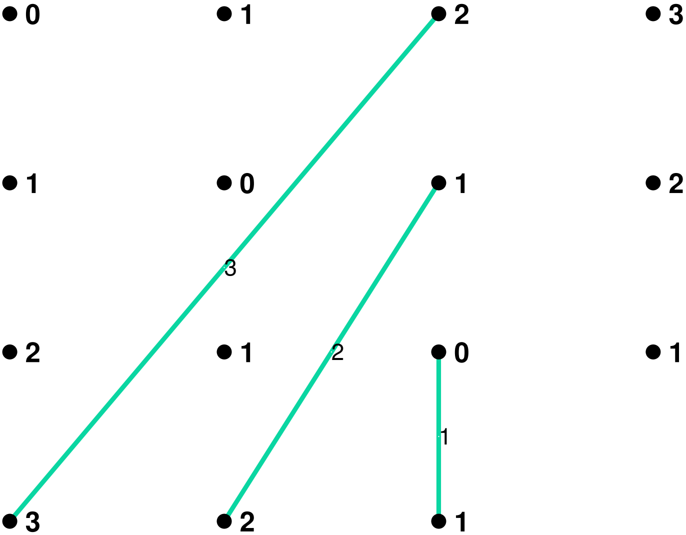
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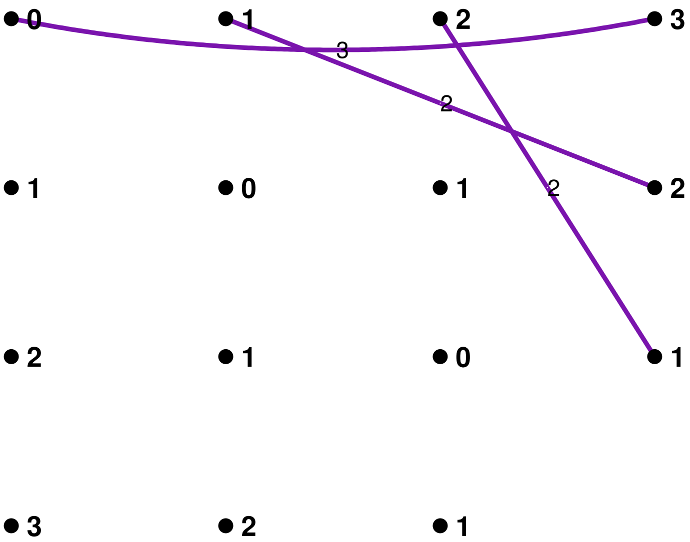
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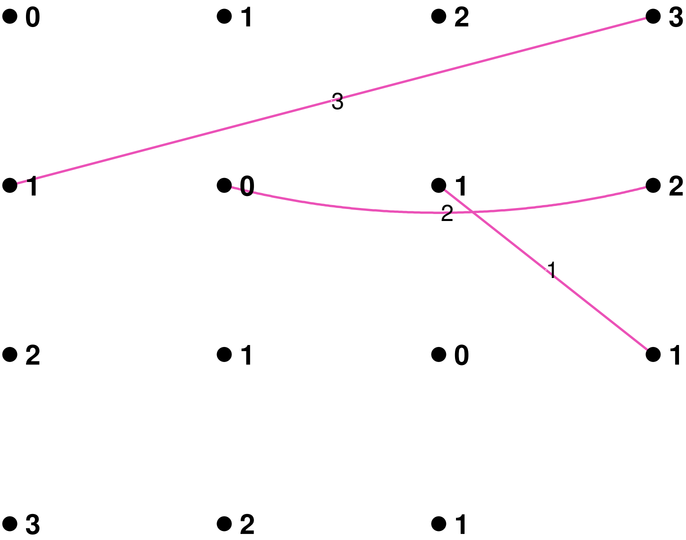

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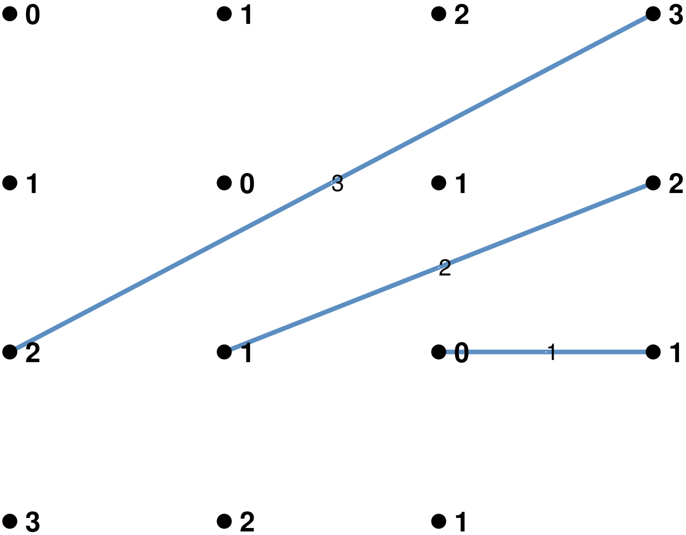
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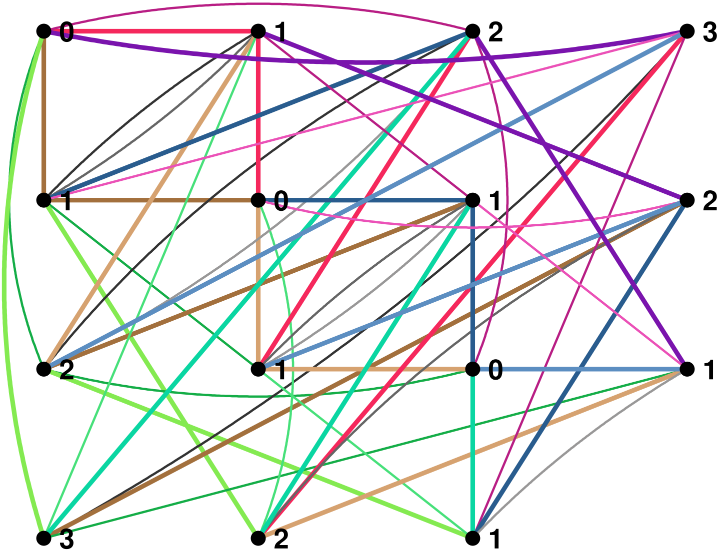
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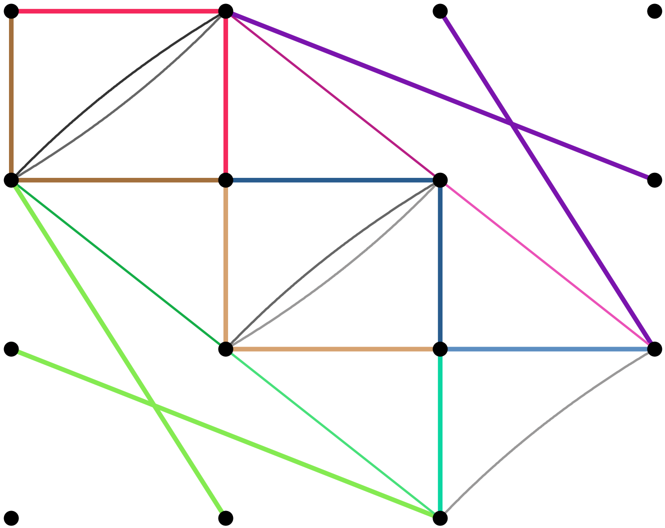

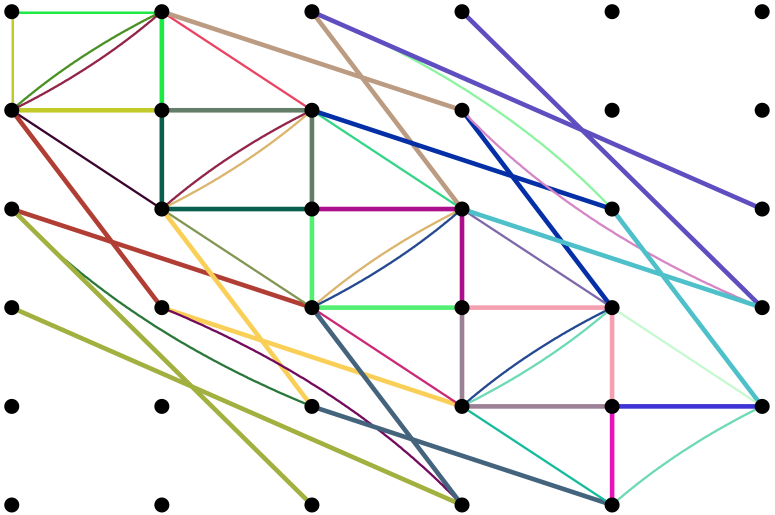

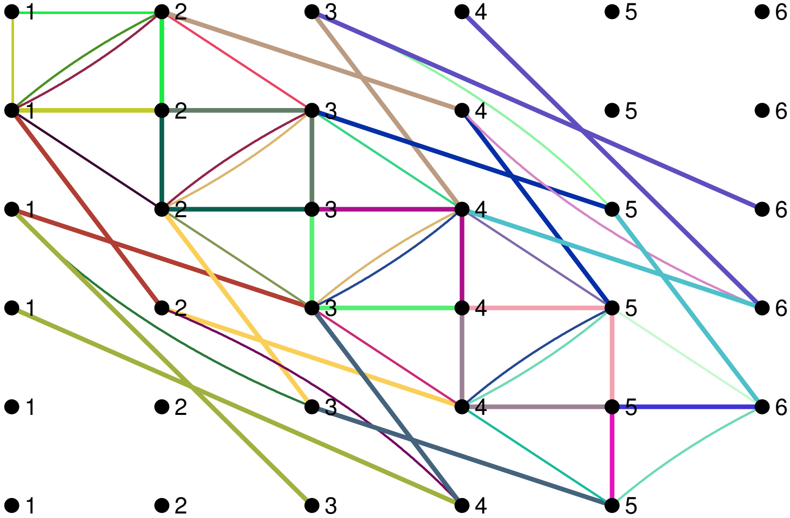
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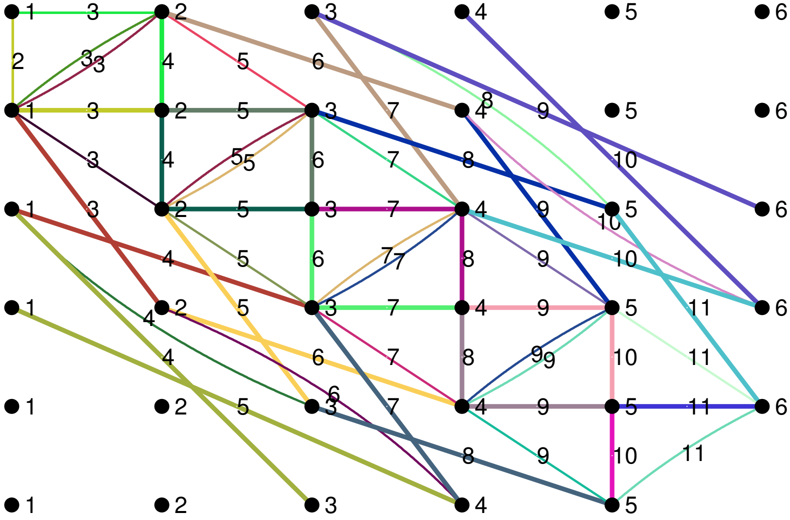

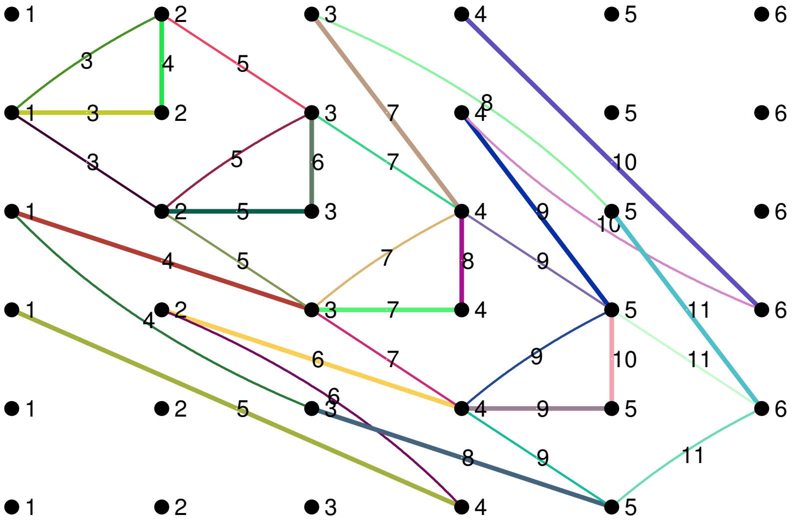

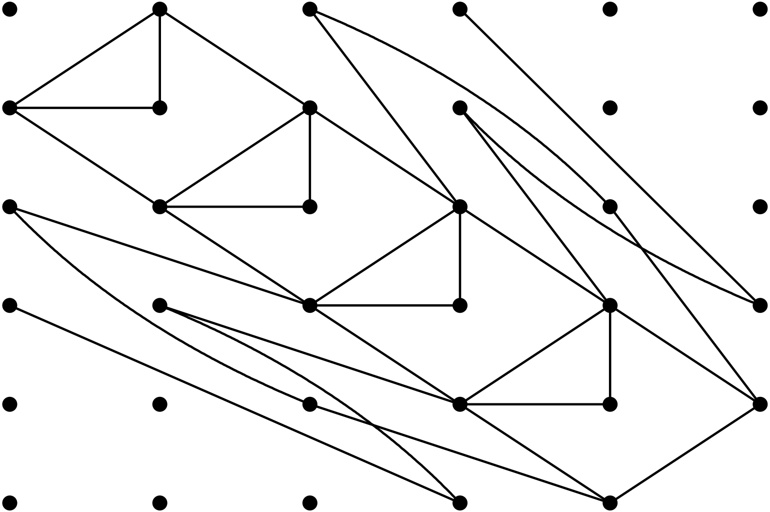

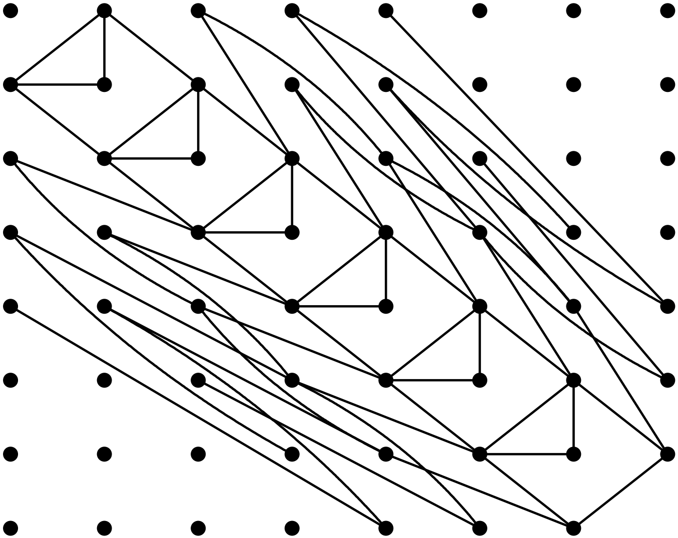

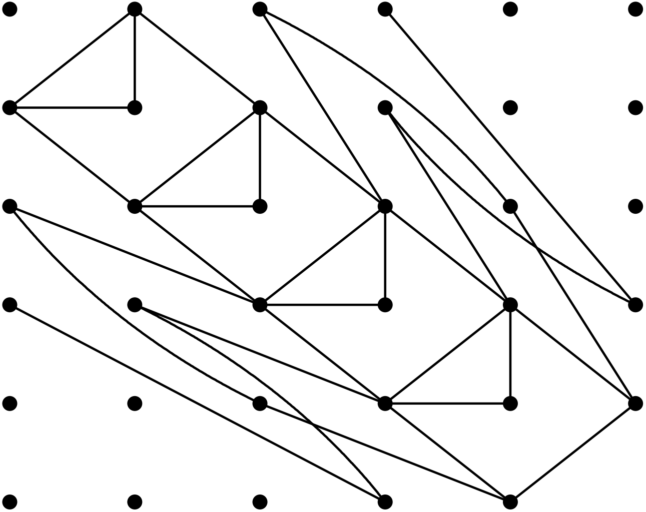

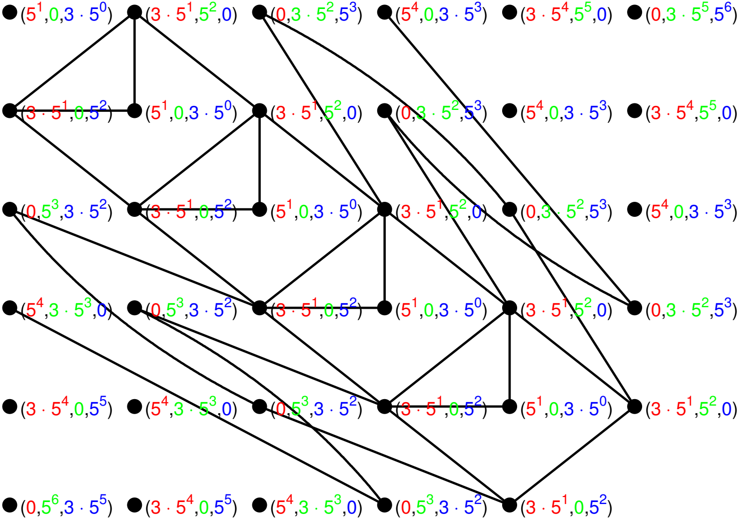

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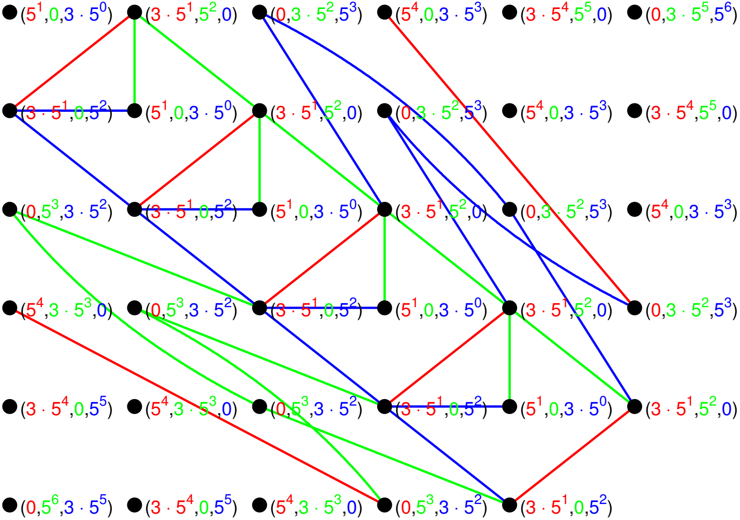
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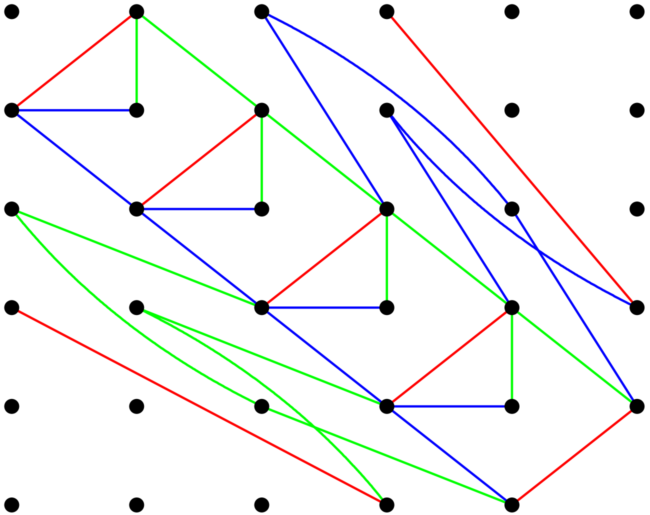

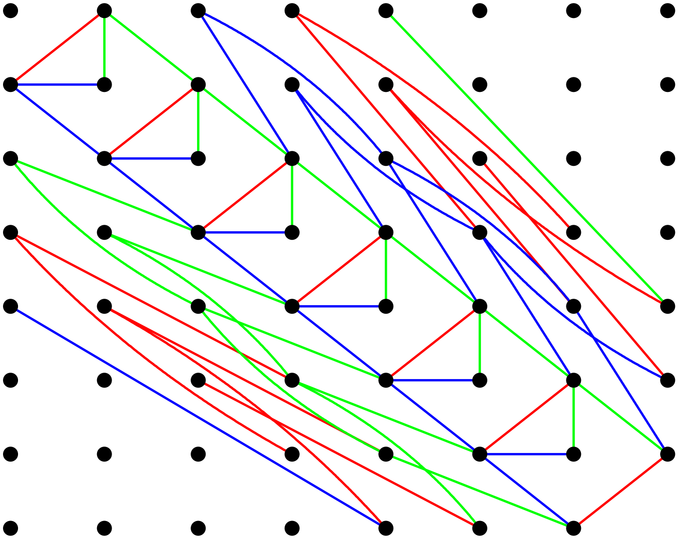

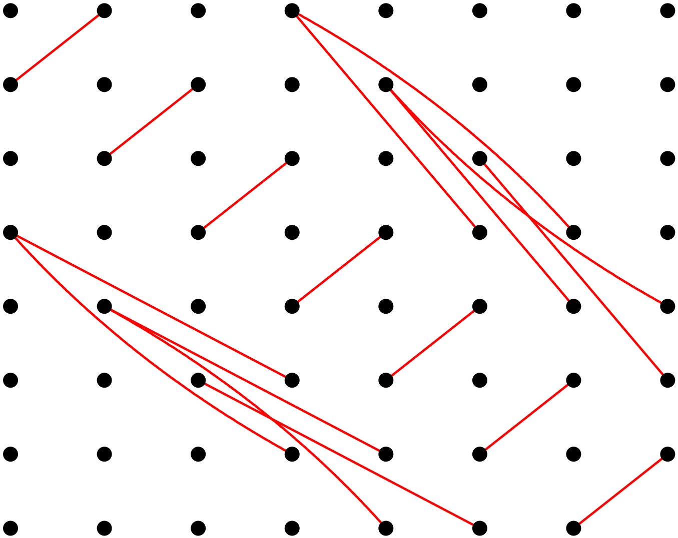

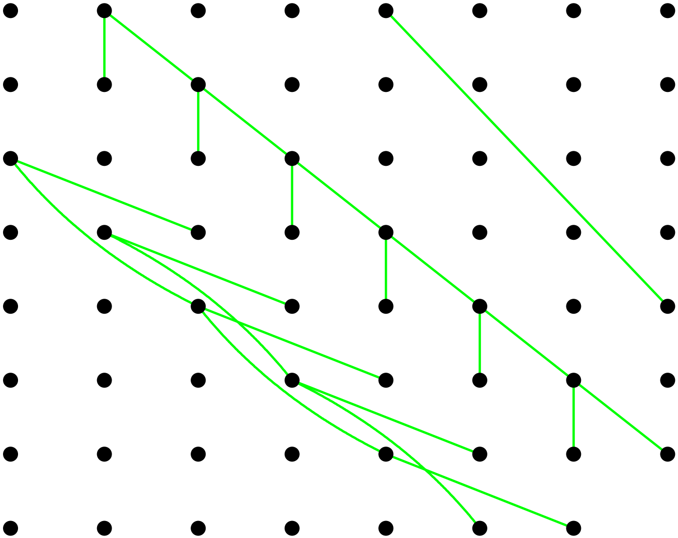

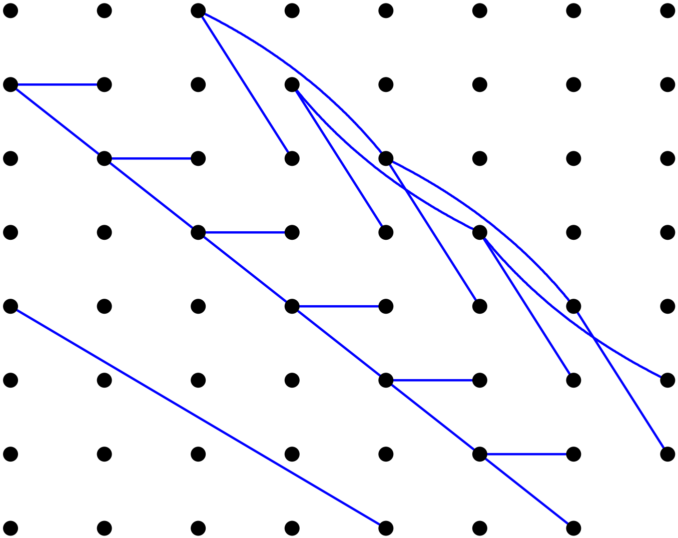

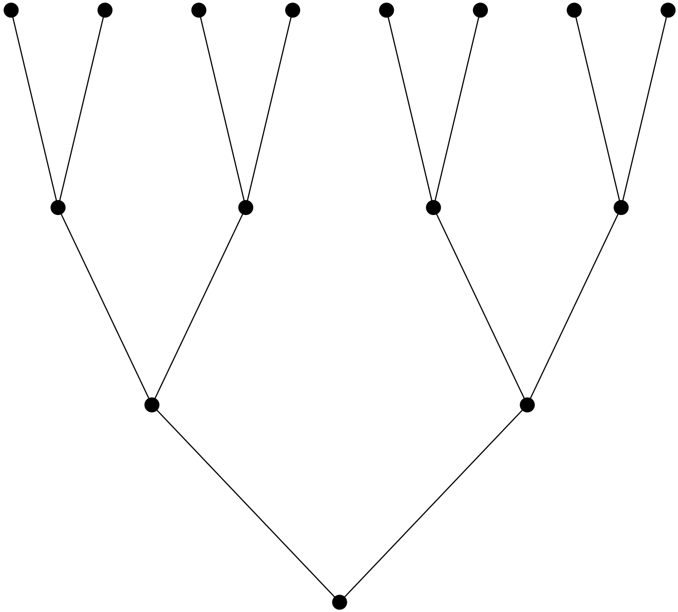

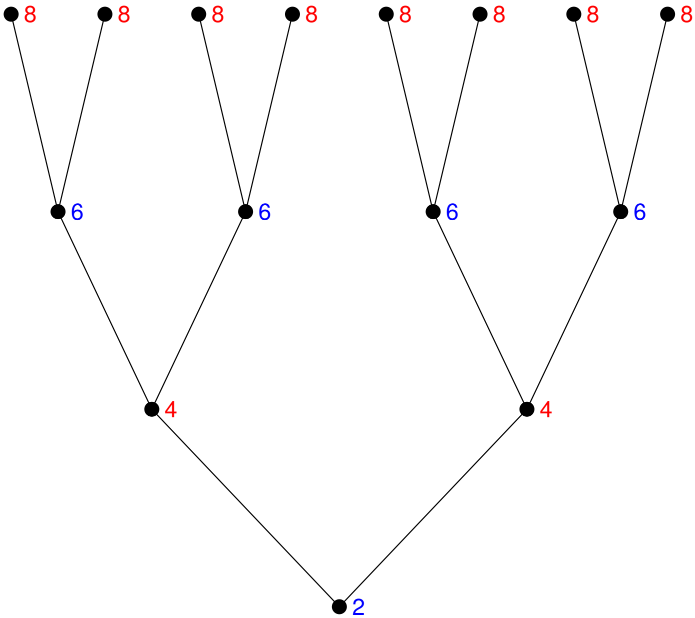
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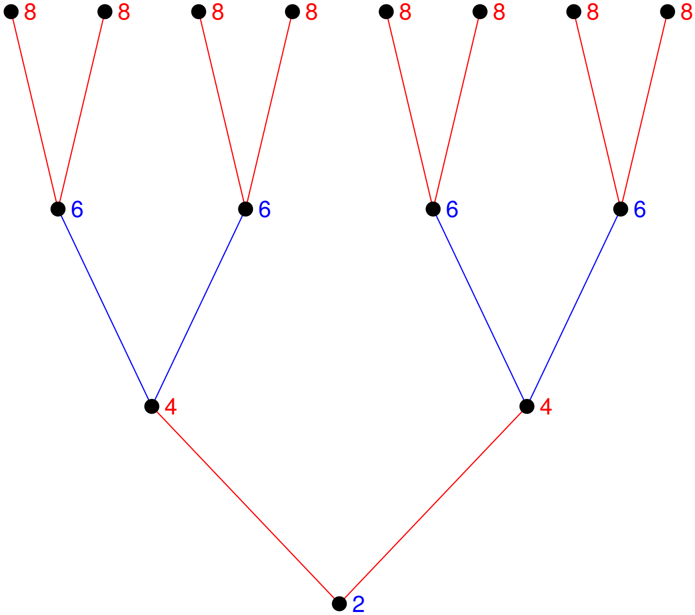
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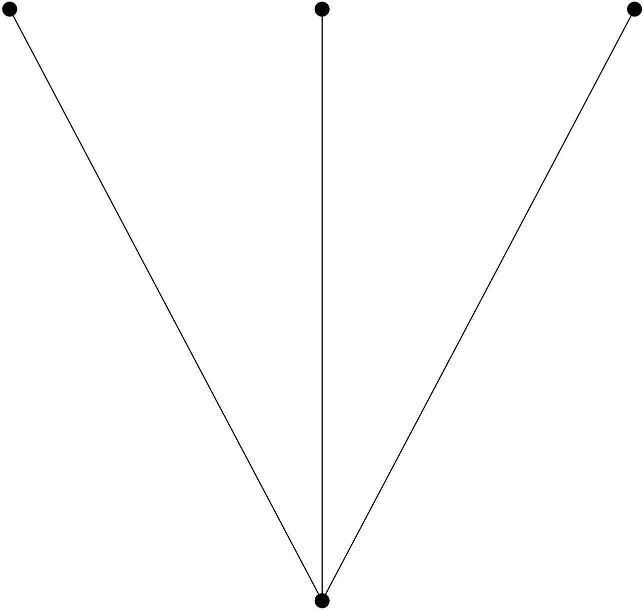

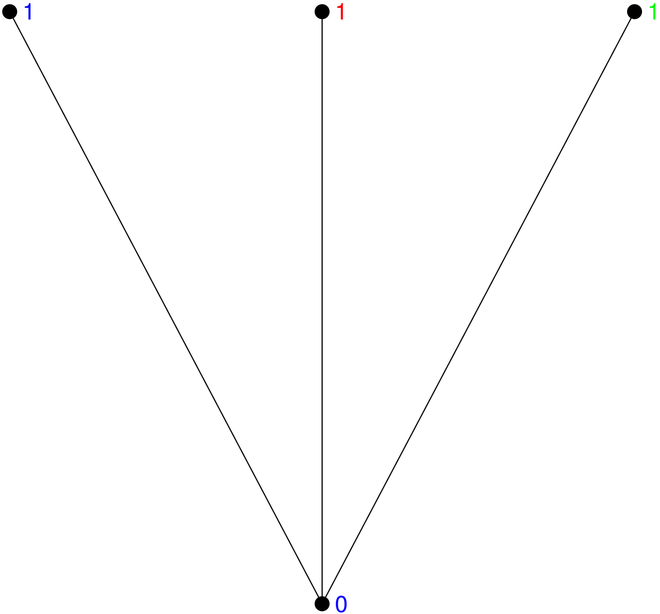
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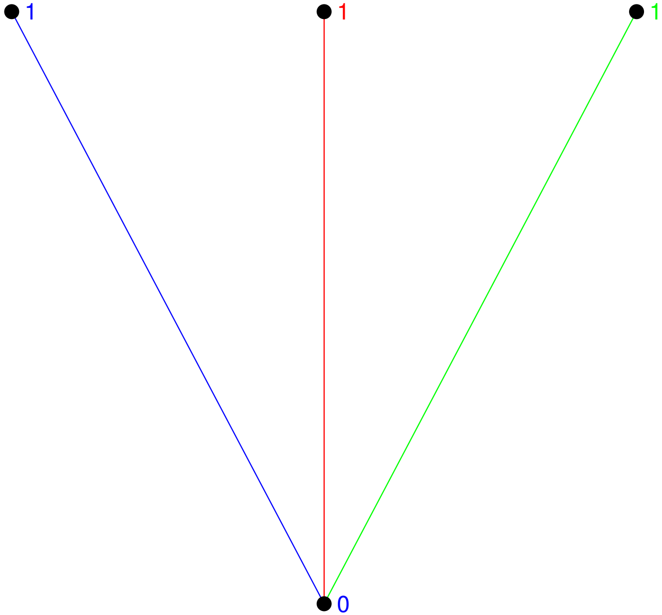
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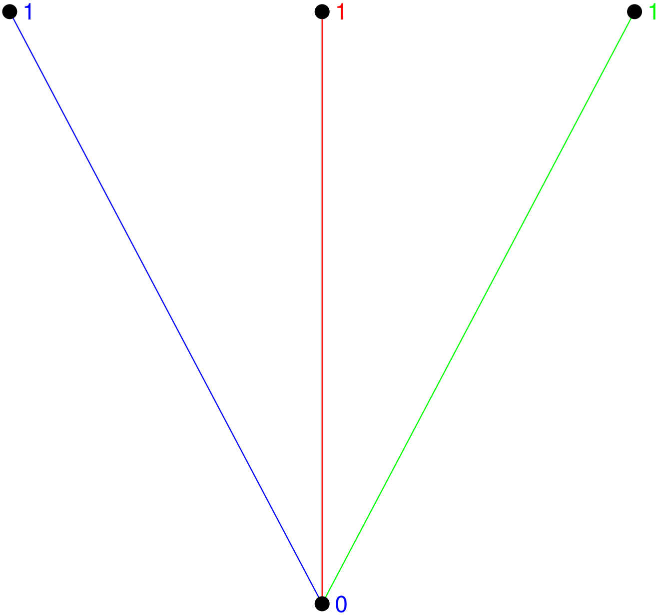
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The continuity criterion

$$\overset{m}{X} \xrightarrow{\phi} Y$$

$X, Y-$ smooth, $\phi-$ FRS , m smooth measure.

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- Problems – There is no simultaneous resolution of singularities. We have no control on the rate of convergence or the values. Not obvious that it will be continuous. One can think of the essence of the result as a quantitative version of Elkik's theorem.

Proof of the continuity criterion

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$$\frac{\phi_*(|\omega_X|f)}{|\omega_Y|}(y) = \int_{(\phi^{-1}(y))^{reg}} |\frac{\omega_X}{\phi^*(\omega_Y)}|f$$

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5. Key lemma – conditions on the monomials: Homological algebra
6. Local computation.

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Then

$\tilde{\omega}/\tilde{f}$ *is regular outside* X_0' *and has a simple pole there.*

Proof of the key lemma

$\mu := \tilde{\omega}/\tilde{f}$ is regular outside X_0' and has a simple pole there.

$$\Uparrow$$

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$$\forall \mu \in \Gamma(\Omega_{\tilde{X}}(\tilde{f})), \mu \in \Gamma(\Omega_{\tilde{X}}(X_0'))$$

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$$Ad(G)$$

$$\circlearrowright$$

$$\begin{array}{ccccc}
G^{2n} & \hookleftarrow & \mathrm{Def} & \xrightarrow{\pi} & \mathrm{Char} := \mathrm{Def}//G \\
\Phi\downarrow & & \downarrow & & \\
G & \ni & 1 & &
\end{array}$$

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Theorem (ess. Witten 1992)

$\omega_{\mathrm{Def}} = \pi^*(\omega^{top}_{\mathrm{Char}}) \otimes \omega_{G,rel}$