

EXERCISE 3 IN D-MODULES I

DMITRY GOUREVITCH AND JOSEPH BERNSTEIN

(1) Exterior products of \mathcal{D} -modules. Consider to affine spaces X, Y . Let M be a $\mathcal{D}(X)$ -module and N be a $\mathcal{D}(Y)$ -module.

(a) Consider the vector space $M \otimes_k N$ and define on it a structure of a $\mathcal{D}(X \times Y)$ -module. This module is called the exterior product of M and N and denoted $M \boxtimes N$.

(b) Prove that

$$d(M \boxtimes N) = d(M) + d(N) \text{ and } e(M \boxtimes N) \leq \binom{d(M) + d(N)}{d(M)} e(M)e(N).$$

(2) Tensor product over \mathcal{O} . Let M, N be $\mathcal{D}(X)$ -modules (where X is an affine space). Consider the space $M \otimes N := M \otimes_{\mathcal{O}(X)} N$ and define the structure of a $\mathcal{D}(X)$ -module by Leibnitz rule.

(a) Show that $M \otimes N$ is canonically isomorphic to the module $\Delta^0(M \boxtimes N)$, where $\Delta : X \rightarrow X \times X$ is the diagonal embedding

(b) Show that the inner tensor product $M \otimes N$ of finitely-generated \mathcal{D} -modules is not always finitely generated.

(3) (P) Show that if M, N are holonomic then $M \otimes N$ is also holonomic.

(4) (P) Let M be a left $\mathcal{D}(X)$ -module and N be a right $\mathcal{D}(X)$ -module. Define a natural structure of a right $\mathcal{D}(X)$ -module on $M \otimes N$. Explain the analytic meaning of this construction. Convince yourself that there is no natural tensor product of right $\mathcal{D}(X)$ -modules.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/Dmod1_3.html