EXERCISE 7 IN D-MODULES I

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Let X be a smooth algebraic variety and $Z \subset X$ a (Zariski) closed subvariety. Let $i : Z \hookrightarrow X$ denote the embedding.

(1) Let X be a smooth affine variety, and $Z \subset X$ be a closed smooth subvariety. As in the lecture, define a functor $i' : \mathcal{M}^r(\mathcal{D}_X) \to \mathcal{M}^r(\mathcal{D}_Z)$ by

$$i'(\mathcal{F}) := \mathrm{Hom}_{\mathcal{D}_X}(\mathcal{D}_{Z\to X}, \mathcal{F}).$$

Show that

(i) For affine X, $i'(M)$ is isomorphic as an $\mathcal{O}_Z$-module to the subspace $\mathrm{Ann}_M I(Z)$ of M consisting of elements annihilated by the ideal $I(Z)$.

(ii) $i' i_0 N \simeq N$ for any $N \in \mathcal{M}(\mathcal{D}_Z)$.

(iii) i_0 is left adjoint to i'.

(2) (P) Let V be a vector space, and M be a $\mathcal{D}_V$-module supported at $\{0\}$. Let $E := \sum x_i \partial_i \in \mathcal{D}(V)$ be the Euler operator. Show that all the eigenvalues of E on M are negative.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/DmodI_3.html

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