Generalized functions - Exercise 3

Solve the following exercises. Questions marked with (*) are optional.

1. Let $W \subseteq V$ be locally convex topological vector spaces, and set $V'$ and $W'$ to be the continuous duals of $V$ and $W$ respectively, and let $(\cdot)'$ denote the full linear dual.

   (i) Show that the restriction map $V' \rightarrow W'$ is onto.

   (ii) Show that the restriction map $V' \rightarrow W'$ is onto.

2. Show that $f_n \in C_c^\infty(\mathbb{R})$ converges to $f$ with respect to the topology defined in class if and only if it converges as was defined in the first lecture, i.e,

   (a) There is a compact set $K \subset \mathbb{R}$ s.t. $\text{supp}(f) \cup \bigcup_{n \in \mathbb{N}} \text{supp}(f_n) \subseteq K$.

   (b) For every $k \in \mathbb{N}$ the derivatives $f_n^{(k)}(x)$ converge uniformly to $f^{(k)}(x)$.

3. (a) Show that the space of distributions $(C_c^\infty(\mathbb{R}))^*$ is not complete w.r.t the weak topology.

   (b) *Show that its completion is the space of all linear functionals (not necessarily continuous) equipped with the weak topology.