Generalized functions - Exercise 3

Solve the following exercises. Questions marked with (*) are optional.

1. Let $W \subseteq V$ be locally convex topological vector spaces, and set V' and W' to be the continuous duals of V and W respectively, and let $()^{\#}$ denote the full linear dual.

   (i) Show that the restriction map $V^{\#} \to W^{\#}$ is onto.

   (ii) Show that the restriction map $V' \to W'$ is onto.

2. Show that $f_n \in C_c^\infty(\mathbb{R})$ converges to f with respect to the topology defined in class if and only if it converges as was defined in the first lecture, i.e,

   (a) There is a compact set $K \subset \mathbb{R}$ s.t. $\operatorname{supp}(f) \bigcup_{n \in \mathrm{N}} \operatorname{supp}(f_n) \subseteq K$.

   (b) For every $k \in \mathbb{N}$ the derivatives $f_n^{(k)}(x)$ converge uniformly to $f^{(k)}(x)$.

3. (a) Show that the space of distributions $(C_c^\infty(\mathbb{R}))^*$ is not complete w.r.t the weak topology.

   (b) *Show that its completion is the space of all linear functionals (not necessarily continuous) equipped with the weak topology.