Generalized functions - Exercise 6

Solve the following exercises. Questions marked with (*) are optional.
  Fix $W \subseteq V$ and E to be finite dimensional topological vector spaces.

1. Show that $C^{-\infty}(V) \otimes E \simeq (C_c^\infty(V, E^* \otimes \mathrm{Haar}(V)))^*$.

2. Define an embedding $C_c^\infty(V, E) \hookrightarrow C^{-\infty}(V, E)$.

3. Show the following,

   (a) $\mathrm{Haar}(V)$ is in canonical isomorphism with $\mathrm{Haar}(W) \otimes \mathrm{Haar}(V/W)$.

   (b) *$\Omega^{\mathrm{top}}(V) \simeq \Omega^{\mathrm{top}}(W) \otimes \Omega^{\mathrm{top}}(V/W)$.

   (c) *$\mathrm{Ori}(V) \simeq \mathrm{Ori}(W) \otimes \mathrm{Ori}(V/W)$.

   (d) $\mathrm{Haar}(V)^* = \mathrm{Haar}(V^*)$.

4. Find a space which is Hausdorff, locally isomorphic to $\mathbb{R}^n$ but is not para-compact.

5. Show that $C^\infty(\mathbb{R}^n, \mathbb{R}^k) = \{f : \mathbb{R}^n \to \mathbb{R}^k : \mu \circ f \in C^\infty(\mathbb{R}^n) \forall \mu \in C^\infty(\mathbb{R}^k)\}$.