Multiplicity One Theorems

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Formulation

Let F be a local field of characteristic zero.

Theorem (Aizenbud-Gourevitch-Rallis-Schiffmann-Sun-Zhu)

Every $GL_n(F)$-invariant distribution on $GL_{n+1}(F)$ is transposition invariant.

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It has the following corollary in representation theory.

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Let π be an irreducible admissible representation of $\mathrm{GL}_{n+1}(F)$ and τ be an irreducible admissible representation of $\mathrm{GL}_n(F)$. Then

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Similar theorems hold for orthogonal and unitary groups.

- $\widetilde{G} := GL_n(F) \rtimes \{1, \sigma\}$
- Define a character χ of $\widetilde{G}$ by $\chi(GL_n(F)) = \{1\}$, $\chi(\widetilde{G} - GL_n(F)) = \{-1\}$.

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By the homogeneity theorem, the stratification method and Frobenius descent we get that any $\xi \in \mathcal{S}^*(X)^{\widetilde{G},\chi}$ is supported in S'.

Singular Support and Wave Front Set

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In the non-Archimedean case we define the singular support to be the Zariski closure of the wave front set.

Properties and the Integrability Theorem

Let X be a smooth algebraic variety.

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- Let an algebraic group G act on X. Let $\xi \in \mathcal{S}^*(X)^{G,\chi}$. Then

$$SS(\xi) \subset \{(x,\phi) \in T^*X \,|\, \forall \alpha \in \mathfrak{g} \quad \phi(\alpha(x)) = 0\}.$$

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- Let V be a linear space. Let $Z \subset V^*$ be a closed subvariety, invariant with respect to homotheties. Let $\xi \in \mathcal{S}^*(V)$. Suppose that $\mathrm{Supp}(\widehat{\xi}) \subset Z$. Then $SS(\xi) \subset V \times Z$.

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- Integrability theorem:
  Let $\xi \in \mathcal{S}^*(X)$. Then $SS(\xi)$ is (weakly) coisotropic.

Coisotropic varieties

Definition

Let M be a smooth algebraic variety and ω be a symplectic form on it. Let $Z \subset M$ be an algebraic subvariety. We call it ***M*-coisotropic** if the following equivalent conditions hold.

- At every smooth point $z \in Z$ we have $T_z Z \supset (T_z Z)^{\perp}$. Here, $(T_z Z)^{\perp}$ denotes the orthogonal complement to $T_z Z$ in $T_z M$ with respect to ω.
- The ideal sheaf of regular functions that vanish on $\overline{Z}$ is closed under Poisson bracket.

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- Every non-empty coisotropic subvariety of M has dimension at least $\frac{\dim M}{2}$.

Weakly coisotropic varieties

Definition

Let X be a smooth algebraic variety. Let $Z \subset T^*X$ be an algebraic subvariety. We call it T^*X-**weakly coisotropic** if one of the following equivalent conditions holds.

- For a generic smooth point $a \in p_X(Z)$ and for a generic smooth point $y \in p_X^{-1}(a) \cap Z$ we have $CN^X_{p_X(Z),a} \subset T_y(p_X^{-1}(a) \cap Z)$.
- For any smooth point $a \in p_X(Z)$ the fiber $p_X^{-1}(a) \cap Z$ is locally invariant with respect to shifts by $CN^X_{p_X(Z),a}$.

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- Every non-empty weakly coisotropic subvariety of T^*X has dimension at least $\dim X$.

Definition

Let X be a smooth algebraic variety. Let $Z \subset X$ be a smooth subvariety and $R \subset T^*X$ be any subvariety. We define **the restriction $R|_Z \subset T^*Z$ of R to Z** by

$$R|_Z := q(p_X^{-1}(Z) \cap R),$$

where $q : p_X^{-1}(Z) \to T^*Z$ is the projection.

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Lemma

Let X be a smooth algebraic variety. Let $Z \subset X$ be a smooth subvariety. Let $R \subset T^*X$ be a (weakly) coisotropic variety. Then, under some transversality assumption, $R|_Z \subset T^*Z$ is a (weakly) coisotropic variety.

Reduction to the geometric statement

Notation

$$T' = \{((A_1, v_1, \phi_1), (A_2, v_2, \phi_2)) \in X \times X \,|\, \forall i, j \in \{1, 2\}$$
$$(A_i, v_j, \phi_j) \in S' \text{ and } [A_1, A_2] + v_1 \otimes \phi_2 - v_2 \otimes \phi_1 = 0\}.$$

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It is enough to show:

Theorem (The geometric statement)

There are no non-empty $X \times X$-weakly coisotropic subvarieties of T'.

Reduction to the Key Lemma

Notation

$$T'' := \{((A_1, v_1, \phi_1), (A_2, v_2, \phi_2)) \in T' | A_1^{n-1} = 0\}.$$

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Notation

Let $A \in sl(V)$ be a nilpotent Jordan block. Denote $R_A := (T' - T'')|_{\{A\} \times V \times V^*}$.

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Lemma (Key Lemma)

There are no non-empty $V \times V^* \times V \times V^*$-weakly coisotropic subvarieties of R_A.

Proof of the Key Lemma

Notation

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$$f(v_1, \phi_1, v_2, \phi_2) := (v_1)_i(\phi_2)_{i+1} - (v_2)_i(\phi_1)_{i+1}.$$

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It is enough to show that $f(R_A \cap L_{ii}) = \{0\}$.

Proof of the Key Lemma

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We know that there exists a nilpotent B satisfying $[A, B] = M$. Hence this B is upper nilpotent, which implies $M_{i,i+1} = 0$ and hence $f(v_1, \phi_1, v_2, \phi_2) = 0$.

Summary

Flowchart

$sl(V) \times V \times V^*$

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$$sl(V) \times V \times V^* \xrightarrow[descent]{H.Ch.} S$$

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sl(V) \times V \times V^* \arrow[r, "H.Ch."', "descent"'] & S \arrow[r, "\text{Fourier transform and}", "\text{homogeneity theorem}"'] & S' \arrow[r, "\text{Fourier transform and}", "\text{integrability theorem}"'] & T' \arrow[d] \\
& & & T' - T''
\end{tikzcd}$$

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\emptyset & L_{ii}\cap R_A \arrow[l, "f(R_A\cap L_{ii})=0"'] & R_A \arrow[l, "R_A\subset\bigcup L_{ij}"'] & T' - T'' \arrow[l, "restriction"']
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