Perverse sheaves and their applications.

Joseph Bernstein **Fall 2016**

Syllabus.

I. Generalities on sheaves
1. General properties of sheaves. Standard functors.
2. Derived categories of sheaves.
3. Functors between derived categories of sheaves.
(i) Push forward and ! push forward
(ii) Pull back and ! pull back
(iii) Verdier duality
4. Standard triangle

II. Sheaves on complex algebraic varieties.
1. Local systems and their properties.
2. Perverse sheaves.
3. Relation with D-modules
4. Functors on perverse sheaves.
5. Properites of the category $Perv$ of perverse sheaves
6. Classification of irreducible perverse sheaves.

III. Functors of nearby cycles and vanishing cycles.

IV. Weil conjectures.
1. Etale topology and comparison theorems.
2. l-adic contructible sheaves. Derived category of l-adic sheaves.
3. Functors.

V. Sheaves over varieties over finite fields.
1. Frobenius.
2. Weil sheaves and Weil complexes.
3. Mixed complexes. Functorrs. Sheaves to functions corespondence.
4. Weight filtration.
5. Main results about weight filtration.

VI. Filtration on perverse sheaves.
1. Purity of irreducible perverse sheaves.
2. Decomposition theorem for pure sheaves.
3. Decomposition theorem for push forward with respect to proper morphisms
6. Constructible sheaves and complexes.
7. Functors between categories of constructible complexes.

II. t-structures
1. t -structures on triangulated categories. Heart of t-structure.
2. Perverse t-structure
3. Case of small and semi-small proper morphisms.
4. Nearby cycles, perversity and filtration.

VII. Applications.

1. Localization theorem and Kazhdan-Lusztig conjectures.

2. Character sheaves

3. Springer resolution

VIII. Equivarint derived categories.

1. Equivariant derived category and equivariant perverse sheaves.

2. Applications to toric varieties

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