# EXERCISE 10 IN INTRODUCTION TO REPRESENTATION THEORY

DMITRY GOUREVITCH

(1) (P) Demonstrate the Brower Induction Theorem for $S_3$. Namely, show how the character of each of the irreducible representations from the classification in section 3 can be expressed as an integer linear combination of inductions of characters of cyclic subgroups (in this case all elementary subgroups are cyclic).

(2) (P) Let $G$ be a finite group and $p$ be a prime number. Show that for any $g \in G$ there exist unique $g_r, g_s \in G$ such that $g = g_r g_s = g_s g_r$, the order of $g_r$ is prime to $p$ and the order of $g_s$ is a power of $p$.

(3) (P) Let $G := SO(2) = S^1$ be the group of rotations of the real plane. It can also be viewed as a circle inside the (complex) plane. Consider the regular representation of $G$ in the Banach space $C(G)$ of continuous functions on $G$ with maximum norm. Let $\mathrm{End}(C(G))$ denote the Banach space of continuous operators from $C(G)$ to itself with the operator norm. Show that the map $G \to \mathrm{End}(C(G))$ given by the representation is not continuous.

(4) ($\square$) Show that the representation of $\mathbb{R}$ in the space of all bounded continuous functions on $\mathbb{R}$ is not a continuous representation.

*URL*: http://www.wisdom.weizmann.ac.il/~dimagur/RepTheo4.html

*Date*: July 17, 2020.