

## EXERCISE 4 IN INTRODUCTION TO REPRESENTATION THEORY

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- (1) For any representation  $\pi$  of a group  $G$ , the following are equivalent:
  - (a)  $\pi$  is isotypic
  - (b) All irreducible subrepresentations of  $\pi$  are isomorphic
  - (c) If  $\pi \simeq \omega \oplus \tau$  with  $\langle \omega, \tau \rangle = 0$  then either  $\omega = 0$  or  $\tau = 0$ .
  
- (2) (P) Classify all irreducible representations of the group  $G$  of motions of a cube.  
*Hint.* Use the action of  $G$  on faces, edges, vertices and main diagonals of the cube, and on regular tetrahedra inscribed in the cube.