EXERCISE 5 IN INTRODUCTION TO REPRESENTATION THEORY

DMITRY GOUREVITCH

(1) Let a (finite) group G act on a (finite) set X. Let π_X be the corresponding representation of G on $F(X)$, and let χ_X be the corresponding character. Show that for any $g \in G$, $\chi_X(g)$ equals the number of elements of X fixed by g.

(2) Show that the character of the regular representation equals $|G|$ at the identity of the group, and equals zero in all other points.

(3) (P) Compute the characters of all irreducible representations of S_4. Use the description we gave on lecture 3. Hint: $\chi_{\pi\oplus\tau} = \chi_\pi + \chi_\tau$.

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