EXERCISE 7 IN INTRODUCTION TO REPRESENTATION THEORY

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(1) Let X be a G-set and $\mathcal{F}$ be a sheaf on X. The following structures on $\mathcal{F}$ are equivalent:
   (a) An equivariant structure
   (b) For any $x \in X$ and $g \in G$ - a linear map $\pi(g)_x : \mathcal{F}_x \to \mathcal{F}_{gx}$ such that for $g_1, g_2 \in G$, $\pi(g_1 g_2)_x = \pi(g_1) \circ \pi(g_2)_x$.

(2) (P) Let $H \subset G$ be a subgroup, and $\pi \in Rep(H)$. Define an isomorphism $\mathcal{I}nd(\pi)(G/H) \cong Ind_H^G(\pi)$.

(3) (P) Let a (finite) group S act on a (finite) commutative group N, and let G be the corresponding semi-direct product $G = S \ltimes N$. Show that for any $\pi \in Irr(G)$, $\dim \pi \leq |S|$.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/IntRepTheo4.html

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