EXERCISE 9 IN INTRODUCTION TO REPRESENTATION THEORY

DMITRY GOUREVITCH

(1) (P) Show that $(G \times G, \Delta G)$ is a Gelfand pair for any group G.

(2) (P) Show that $(S_{n+2}, S_n \times S_2)$ is a Gelfand pair.

(3) (P) Show that (S_4, S_2) is not a Gelfand pair.

(4) (P) Let A be the commutative algebra consisting of all matrices of the form $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$. Show that A has only one simple module M. This module is one-dimensional, and the nilpotent element $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ acts on it by zero.