EXERCISE 5 IN INTRODUCTION TO REPRESENTATION THEORY - INDUCTION

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(1) (a) $H = \{e\}$, $\mathrm{Ind}_H^G(\mathbb{C}) = F(G)$.
(b) For any $H, \mathrm{Ind}_H^G(\mathbb{C}) = F(G/H)$.
(c) For any character χ of H, $\mathrm{Ind}_H^G(\chi) = \{f \in F(G) : f(gh) = \chi(h^{-1})f(g)$.

(2) (a) For $H < G$ and $\pi_1, \pi_2 \in Rep(H)$,
$$\mathrm{Ind}_H^G(\pi_1 \oplus \pi_2) = \mathrm{Ind}_H^G(\pi_1) \oplus \mathrm{Ind}_H^G(\pi_2).$$
(b) For $H_1 < H_2 < G$ and $\pi \in Rep(H)$,
$$\mathrm{Ind}_{H_2}^G \mathrm{Ind}_{H_1}^{H_2} \pi = \mathrm{Ind}_{H_1}^G \pi$$

(3) (P) Let G be a finite group, D its subgroup and χ a character of D. Consider the induced representation $\pi = Ind_D^G(\chi)$. Show that π is irreducible iff the following condition holds:
(*) For any $g \in G$ with $g \notin D$ there exists an element $x \in D$ such that the element $y = gxg^{-1}$ belongs to D and $\chi(x) \neq \chi(y)$.

(4) (P) Barak has got an advanced game, where a usual game cube was replaced by an dodecahedron with numbers $1, ..., 12$ on its faces. Each time he lost, he replaced the number on each face by the average of its neighbors. What numbers will be written on the faces after 30 losses? What is the precision of your answer?

(5) (*) Can you give a definite answer if we replace the cube by an icosahedron?

URL: http://www.wisdom.weizmann.ac.il/~dimagur/IntRepTheo5.html

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