EXERCISE 7 IN INTRODUCTION TO REPRESENTATION THEORY

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(1) Show that any subgroup and quotient group of a c-solvable group is c-solvable. Show that any finite nilpotent group is c-solvable.

(2) (P) Suppose we know that a group G has a commutative normal subgroup N such that the group G/N is c-solvable. Show that any irreducible representation σ of G is monomial.

(3) (P) Let a (finite) group S act on a (finite) commutative group N, and let G be the corresponding semi-direct product $G = S \ltimes N$. Show that for any $\pi \in Irr(G)$, $\dim \pi \leq |S|$.

(4) (P) Let G be a finite group, Z its central subgroup and χ a character of Z. Denote by $Irr(G)_\chi$ the set of equivalence classes of irreducible representations of G on which Z acts via the character with the central character χ.
   (a) Compute $\sum_{\sigma \in Irr(G)_\chi} \dim^2 \sigma$.
   (b) Explain how to find the size of the set $Irr(G)_\chi$. In particular show that this size is maximal when χ is a trivial character.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/RepTheo5.html

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