(1) Show that any subgroup and quotient group of a c-solvable group is c-solvable. Show that any finite nilpotent group is c-solvable.

(2) (P) Suppose we know that a group $G$ has a commutative normal subgroup $N$ such that the group $G/N$ is c-solvable. Show that any irreducible representation $\sigma$ of $G$ is monomial.

(3) (P) Let a (finite) group $S$ act on a (finite) commutative group $N$, and let $G$ be the corresponding semi-direct product $G = S \rtimes N$. Show that for any $\pi \in Irr(G)$, $\dim \pi \leq |S|$.

(4) (P) Let $G$ be a finite group, $Z$ its central subgroup and $\chi$ a character of $Z$. Denote by $Irr(G)_\chi$ the set of equivalence classes of irreducible representations of $G$ on which $Z$ acts via the character with the central character $\chi$.

(a) Compute $\sum_{\sigma \in Irr(G)_\chi} \dim^2 \sigma$.

(b) Explain how to find the size of the set $Irr(G)_\chi$. In particular show that this size is maximal when $\chi$ is a trivial character.