The Categorical Weil Representation and the Sign Problem

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Abstract: I will explain a solution obtained with Ofer Gabber to the sign problem formulated by Bernstein and Deligne. Given a symplectic vector space (V, ω) over $k = \mathbb{F}_q$ and an additive character $\psi : k \to \mathbb{C}^*$, one can construct a "quantum" Hilbert space $\mathcal{H}(L, \psi)$ attached to a Lagrangian subspace $L \subset V$. In a previous work Hadani and myself constructed a system of Fourier Transforms

$$FT_{M,L} : \mathcal{H}(L, \psi) \to \mathcal{H}(M, \psi),$$

for every pair of Lagrangians (L, M), such that the following multiplicativity condition holds:

$$FT_{N,M} \circ FT_{M,L} = FT_{N,L},$$

for every triples (L, M, N) of Lagrangians. This can be used to construct a canonical model for the Weil representation of the symplectic group $Sp(V, \omega)$. In the lecture I will explain what is the categorification of these Fourier transforms, what is the related sign problem, and what is its solution.