p-adic-10

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10. Lecture 10. Norm fields

K p-adic field.

$G_K = Gal(\bar{K}/K)$.

$Rep(G_K)$

$K \subset K_\infty \subset \bar{K}$, $K_\infty = K(\mu_{p^n} | n)$.

$H = Gal(\bar{K}/K_\infty)$, $\Gamma = Gal(K_\infty/K)$.

$\Gamma \sim \mathbb{Z}_p$

$1 \to H \to G_K \to \Gamma \to 1$

claim. $\exists$ a field $F(K)$ of char. p s.t. $Gal(F(K)_{sep}/F(K)) = H$.

$E((\pi)) \subset F(K)$

claim. $H = Gal(E(K)_{sep}/E(K))$

Let L : field ext. of K,

$K \subset L \subset \bar{K}$

v

$K_\infty \subset M \subset \bar{K}$

$F(K_\infty) \subset F(M) \subset F(\bar{K})$

$F(L) = R$

1. For any extension $L \supset K$ we define a field $R(L)$ of char. p.

$F(L) = R(L_\infty)$, $L_\infty = L(\mu_{p^\infty})$.

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construction of R.

Let L be an extension of K algebraic.

v - valuation on L

$\mathcal{O}_L$ - ring of integers.

$\mathcal{O}_L \supset p\mathcal{O}_L$

$A = \mathcal{O}_L / p\mathcal{O}_L$

$A \xleftarrow{p} A \xleftarrow{p} A \xleftarrow{p} A$

($R(L) = \varprojlim A$

Claim. Let us fix ideal $\mathfrak{a} \subset \mathcal{O}_L$ s.t.

(i) $\mathfrak{a} \supset p\mathcal{O}_L$

(ii) $\mathfrak{a} \sim p^{\mathcal{O}_L}$

$A = \mathcal{O}_L / \mathfrak{a}$

$\varprojlim A$

Proposition. Suppose L is complete w.r.t. v. Then $R(L) = \varprojlim \mathcal{O}_L$,

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