**Complexity Lower Bounds – following Gowers' blog** Fall 2009

# Problem Set # 1

*Due: December 15th, 2009* *Lecturer: Irit Dinur*

General Instructions:

- Please submit the exercise in class, or in Irit's mailbox (Ziskind 2nd floor).
- Try to solve each problem first without consulting references. If you need references, please indicate clearly which reference you used.
- Team work: Allowed, but please limit yourself to groups of at most 3.

Problems:

1. Let $f : \{0,1\}^n \to \{+1,-1\}$. Prove that
$$||f||_{U^2}^4 = 2^{-n} \sum_S |\hat{f}(S)|^4$$
2. Let $f : \{0,1\}^n \to \{\pm 1\}$ be a random function. Prove that with high probability
$$\forall S \ \ |\hat{f}(S)| = O(n2^{-n/2}).$$
3. Use questions 1,2 to deduce that for a random function $||f||_{U^2} \approx 2^{-n/4}$.
4. (*) What is $||f||_{U^k}$ for a random function $f$ ? (i.e. find upper and lower bounds)
5. Let $f, g : \{0,1\}^n \to \{\pm 1\}$ and define $h : \{0,1\}^{2n} \to \{\pm 1\}$ by
$$h(x_1, \ldots, x_n, y_1, \ldots, y_n) = f(x_1, \ldots, x_n)g(y_1, \ldots, y_n)$$
Prove that
$$||h||_{U^k} = ||f||_{U^k} \cdot ||g||_{U^k}$$